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by

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PREDICTION OF SAILING BOAT PERFORMANCE

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ABSTRACT

Mathematical models are developed for the hydrodynamic and aerodynamic forces which act on a sailing boat. The solution of the force and moment equilibrium provides the sailing boat's speed and attitude for performance prediction. Each force model is evaluated for its capability to make adequate engineering predictions. Further, expressions are given for the dimensions and coefficients to provide a feasible baseline design to which parametric variations may be made.

The system of force equations constitute a parameterized system model in four degrees of freedom for steady state sailing conditions. The mathematical model provides a design tool for the naval architect conducting geometric parameter trade-off analysis against hydrodynamic performance.

The principal result is a computerized model which is capable of predicting boat speed, heel angle, leeway angle and rudder angle for a geometrically defined sailing boat in a given wind condition. Example boats are evaluated and the results recorded in order to demonstrate the use and versatility of the model.

Thesis Supervisor: Jerome H. Milgram
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NOMENCLATURE

A_k	=	keel cross-sectional area.
A_l	=	projected lateral area of the hull above the LWL.
A_m	=	cross-sectional area of the maximum hull station, or cross-sectional area of mast depending on context.
A_r	=	platform area of the rudder.
A_s	=	total area of the sails.
A_t	=	projected transverse area of the hull above the LWL.
BWL	=	maximum waterline beam.
C_{dc}	=	rudder cross flow drag coefficient.
C_{do}	=	rudder section form drag coefficient.
C_{dis}	=	sail induced drag coefficient.
C_f	=	friction drag coefficient.
C_{ffb}	=	fairbody friction drag coefficient.
C_{fk}	=	keel friction drag coefficient.
C_{fm}	=	hull form drag coefficient.
C_{fr}	=	rudder friction drag coefficient.
C_{lr}	=	rudder lift coefficient.
C_{ls}	=	sail lift coefficient.
$C_{m\bar{c}/4}$	=	rudder moment coefficient about the mean quarter chord.
C_n	=	rudder normal force coefficient.
C_p	=	prismatic coefficient.
C_{po}	=	parent prismatic coefficient.
C_{popt}	=	optimum prismatic coefficient.
C_{xr}	=	rudder drag coefficient.

C_y = keel side force coefficient

C_{ysb} = slender-body side force coefficient.

C_{yr} = rudder side force coefficient.

D_s = sail drag

D_{so} = offwind sail drag.

F_h = aerodynamic hull force.

F_n = Froude Number.

H = maximum canoe body draft.

K = heeling moment.

K_h = hydrostatic righting moment

K_k = keel heeling moment.

K_r = rudder heeling moment.

K_s = sail heeling moment.

L = load waterline length.

L_s = sail lift.

L_{so} = offwind sail lift.

LCB = position of center of buoyancy (%LWL).

LCB_o = parent position of center of buoyancy.

LCB_{opt} = optimum location of center of buoyancy.

LWL = load waterline length.

M = pitching moment

N = yawing moment

N_k = keel yawing moment.

N_r = rudder yawing moment.

N_{sb} = slender-body yawing moment.

N_{sx} = yawing moment due to sail driving force.

N_{sy} = yawing moment due to sail side force.

$\bar{P}$ = vector of assumed variable values.

$P_{1,2,3,\text{ or }4}$ = assumed value of individual variable.

R_f = total frictional resistance.

R_{fm} = total form resistance.

R_n = Reynold's Number.

R_u = upright resistance.

R_w = wave resistance.

R_{wp} = parent wave resistance.

$\bar{R}$ = vector of force equilibrium errors.

$R_{1,2,3,\text{ or }4}$ = individual force equilibrium errors.

S_{fb} = fairbody wetted surface.

S_k = keel wetted surface.

S_r = rudder wetted surface.

T_c = canoe body maximum draft.

U_b = boat velocity in the X_o direction.

V_a = apparent wind velocity.

V_b = boat velocity parellel to the y axis.

V_t = true wind velocity.

X = force in the X_o direction.

X_h = hull resistance.

X_k = keel and hull induced drag.

X_o = axis on the plane of the free surface in the direction of boat velocity.

X_r = rudder induced drag.

X_s = sail driving force.

Y = force in the Y_o direction, side force.

Y_k = keel side force.

Y_o = an axis on the plane of the free surface at right angles to boat velocity.

Y_r = rudder side force.

Y_s = sail side force.

Z = force in the Z_o direction, heave force.

Z_o = an axis perpendicular to the free surface downward.

a_f = fortriangle aspect ratio.

a_m = mainsail aspect ratio.

a_r = rudder aspect ratio.

a_s = total sail area aspect ratio.

b_b = draft at forward perpendicular.

b_o = maximum draft.

b_r = rudder span.

$b_{1,2,3}$ = intermediate draft measurements.

c_{rr} = rudder root chord.

c_{rt} = rudder tip chord.

$c_{r/4}$ = rudder quarter chord.

$\bar{c}_k$ = keel mean chord.

$\bar{c}_r$ = rudder mean chord.

d_1 = metacentric height/LWL.

d_2 = non-linear hydrostatic righting moment coefficient.

d_3 = Froude Number dependent righting moment coefficient.

$f_i(\ )$ = spline cubic function.

g = acceleration due to gravity.

r_o = canoe body draft at maximum draft measurement.

s = sinkage.

s_{ij} = $\partial R_i/\partial P_j$.

t_r = taper ratio.

x = axis in the load waterline plane along the centerline of the boat.

x_b = horizontal distance to the maximum draft measurement.

x_{cph} = horizontal distance to the aerodynamic center of effort on the hull.

x_{cpk} = horizontal distance to center of effort on the keel.

x_{cpr} = horizontal distance to the center of effort on the rudder.

x_{cps} = horizontal distance to center of effort on the sail.

x_e = main boom length.

x_j = base of the fortriangle.

x_m = horizontal distance to the mast.

x_o = 0.0, location of maximum draft.

$x_{p\bar{c}/4}$ = horizontal distance from the rudder mean chord quarter point to the rudder center of effort.

x_{rs} = horizontal distance to the rudder stock.

$x_{1,2,3}$ = horizontal distance from maximum draft to intermediate draft measurements.

y = axis orthaginal to the x axis to starboard in the waterline plane.

z = axis orthaginal to the waterline plane, downward.

z_{cpk} = vertical distance to the center of effort on the keel.

z_{cpr} = vertical distance to the center of effort on the rudder.

z_{cpxs} = vertical distance to the sail driving force.

z_{cpys} = vertical distance to the sail side force.

z_{fb} = freeboard.

z_g = vertical distance to the center of gravity.

z_i = mast height.

z_p = main hoist.

z_{ro} = vertical distance from the rudder root chord to the center of effort on the rudder.

z_{rrc} = vertical distance to the rudder root chord.

∇_c = volumetric displacement of the fairbody.

∇_k = volumetric displacement of the keel.

Δ = weight of the boat in tons.

α_r = rudder angle.

β = apparent wind angle.

β_u = upright apparent wind angle.

γ = true wind angle.

δ = variable increment.

θ = pitch angle.

λ = angle between boat centerline and aerodynamic hull force.

γ_{H_2O} = kinematic viscosity of water.

π = 3.14159

ρ_{air} = density of air.

ρ_{H_2O} = density of water.

ϕ = heel angle.

ψ = leeway angle.

Ω = rudder sweep angle.

I. INTRODUCTION

1.1 General

All designers of engineering systems desire a procedure for predicting the performance of their designs prior to final construction. Until lately, prediction of a sailing yacht's performance has been limited to evaluation of scale models and intuition. The development of the high speed computer and numerical techniques for rapid solution of non-linear equations has made the use of mathematical models a reasonable alternative for performance prediction. It is the purpose of this paper to present mathematical models for the forces and moments acting on a sailing boat. The solution of the force and moment equilibrium will provide the boat's speed and attitude for performance prediction.

Each force is parameterized in dimensions and coefficients, generally utilized by designers, to make the model a useful design tool for the naval architect. Further, expressions are given for the required dimensions and coefficients to provide a feasible baseline design from which parametric variations may be made. Each force model is evaluated for its capability to make adequate engineering predictions, including inherent limitations.

1.2 Historical Development

From the earliest days, yacht design has been called an "art". Though the use of wind propulsion for ships is ancient, the action of the forces and moments on the sailing ship is

complex and multidimensional. Historically there has been no technique for simple evaluation of the effect a change in one parameter has on the forces acting on the boat.

Until fifty years ago design evolved slowly through obervation of existing ships. Due to the inability to evaluate a change prior to incorporation into a newly constructed yacht, new designs tended to consist of only conservative changes to a previously successful design. This was understandable for failure of an innovation in the final yacht could be professionally catastrophic for the designer.

In the early 1900's, techniques were developed for predicting yacht performance from model tests. A designer could now build a relatively inexpensive model and evaluate it against other model results prior to incorporating a modification into a new design. Adequate prediction of full scale results have been limited by adverse scale effects, but the usefulness of these tests is unquestioned. Unfortunately, the cost of model testing still precludes extensive parametric trade-offs. Each parametric change requires construction and testing of individual models. Usually, the limited design budget for one yacht precludes any systematic variation of parameters, and testing is limited to one or two candidate models. The appeal of a parameterized mathematical model which is inexpensive to utilize is apparent.

The idea of using mathematical expressions for describing yacht performance is by no means a new concept. The rating rules used to handicap yachts are an attempt to apply mathematical models to boat performance. Because rating rules attempt to measure a boat's speed potential for a large range of conditions and reduce the results to a single number, it is easily understood why they have little application to predicting performance for a given wind direction and speed.

It was not until the second half of this century that towing tank test results were combined with sail force coefficients in a computer to solve force and moment equilibriums. These programs could provide prediction of performance for a given condition within the limitations of the data. These solutions were for the specific boat tested and a scaled family of geometrically similar boats.

In the early 1970's attempts for steady state solution from mathematical models began to be published. One pioneer in the field has been John S. Letcher, publishing parts of his model in references (1), (2), and (3). Further work has been done by Myers (4) and Kerwin (5). The computational technique for solution of the force equilibrium condition has been described by Hazen (6).

By parameterizing forces in four degrees of freedom, the model developed in this paper grows from these previous works.

1.3 Overview

In order to resolve the aerodynamic, hydrodynamic and hydrostatic forces acting on a sailing boat a coordinate system and origin are established. Achieving an equilibrium of forces and moments requires that the summation of all forces and moments acting either through the origin and about the origin are equal to zero. To facilitate modeling, component forces which act independently or nearly so are identified and examined. Once the forces acting on the system and the location and direction through which they act are identified, each may be modeled and superimposed to give the required equilibrium solution.

1.3.1 Coordinate System

Figure 1 presents a coordinate system which describes the sailing boats velocity and attitude. The coordinate system designated by the capital letters X_o, Y_o, and Z_o are located on the free surface with X_o and Y_o axis in the horizontal plane. The X_o axis lies in the direction of the boat's motion and the Y_o axis is to starboard at right angles to this velocity. The Z_o axis is vertically downward.

A second coordinate system is fixed in the boat at the forward end of the waterline as presented in Figure 2. This coordinate system is designated by the small letters x, y, and z. The x axis is oriented along the fore and aft center-line on the boat's waterline with the y axis in the transverse

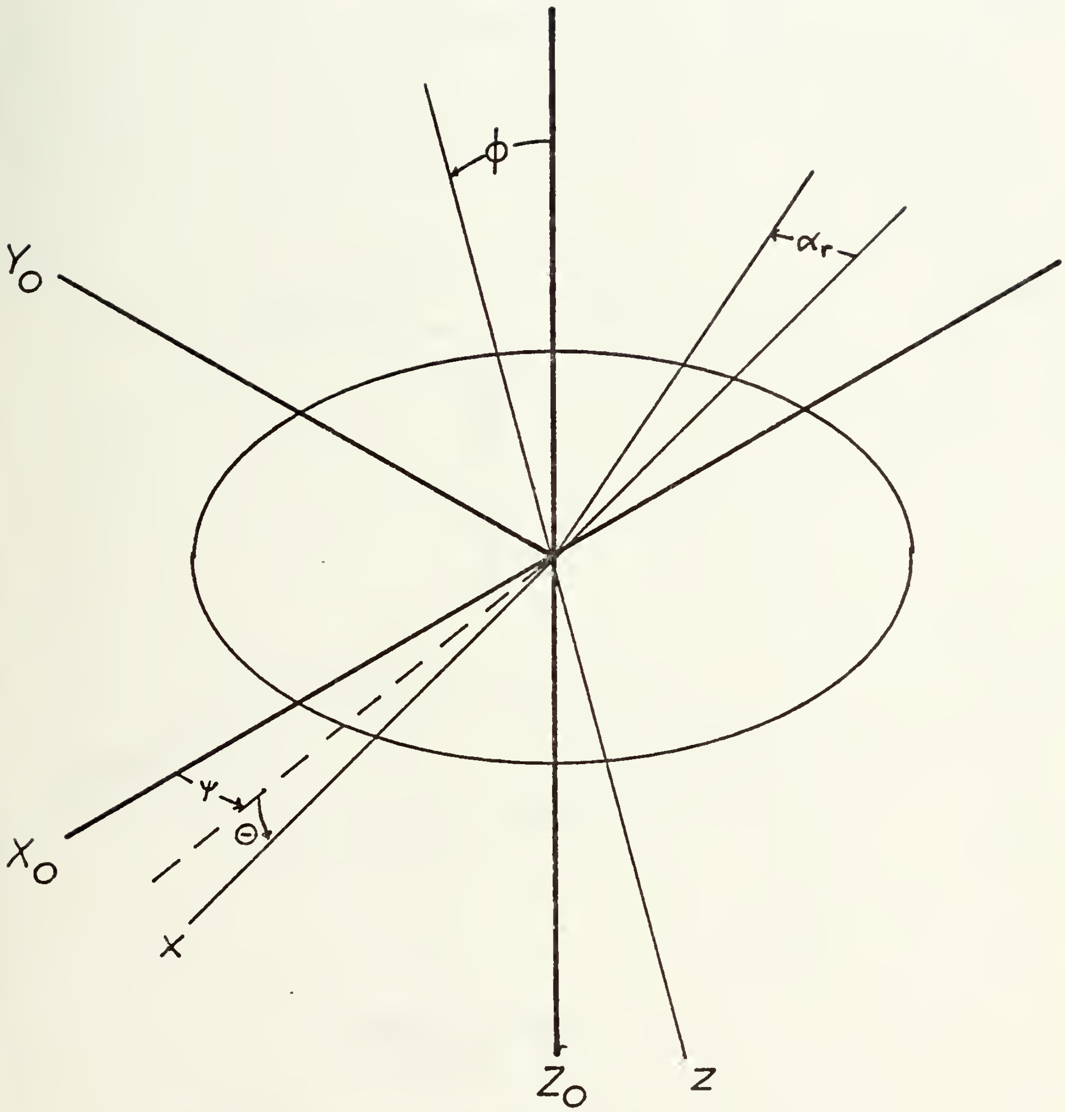

Figure 1 - Coordinate Systems

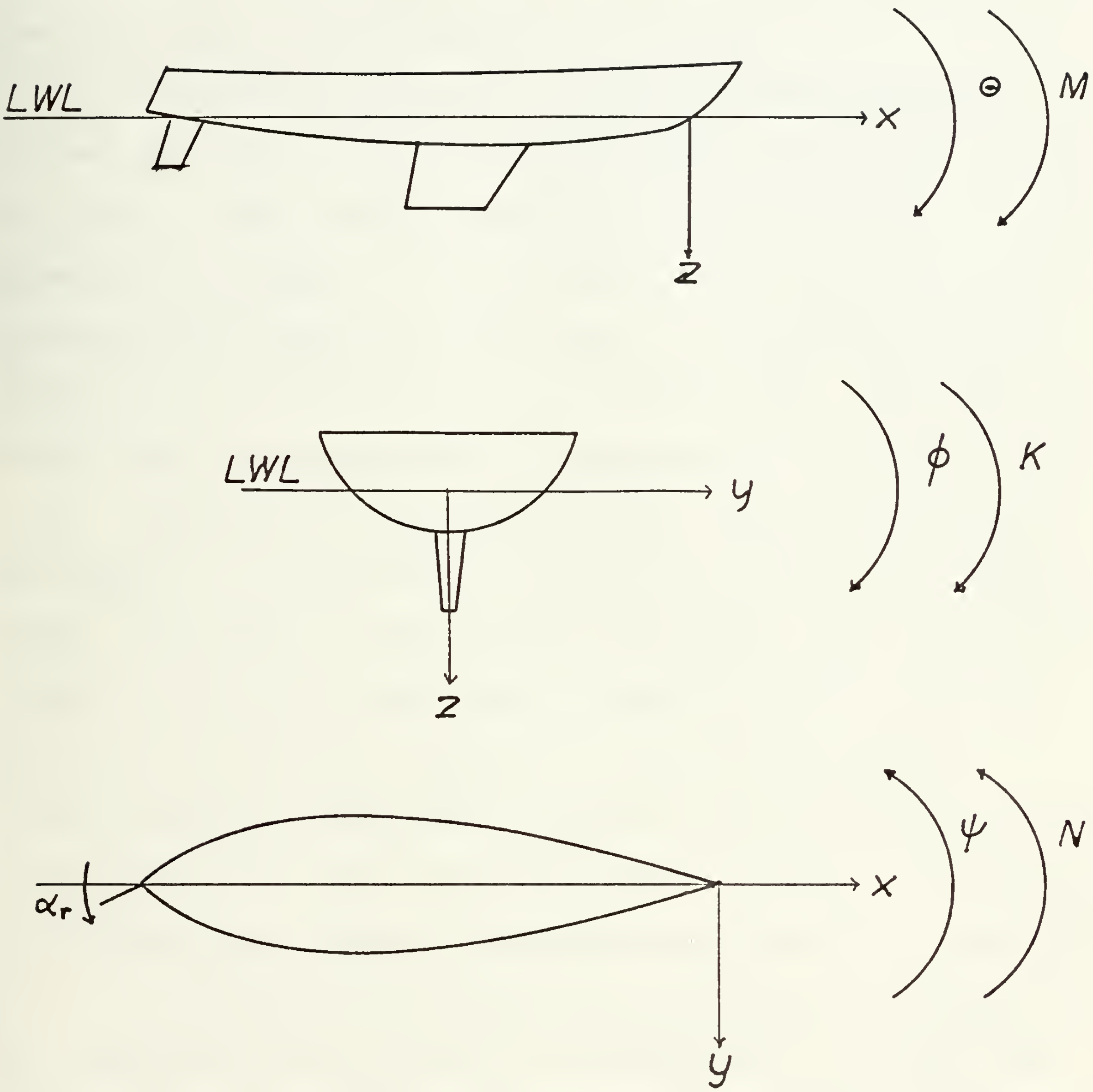

Figure 2

Boat Coordinate System

direction to starboard. The z axis desends orthogonally to the x-y plane in the direction of the keel.

The angular differences between the two coordinate systems describe the attitude of the boat as it moves through the water. The positive direction for these angles are also presented in Figure 2. Their direction was selected to provide positive values when the boat is in equilibrium under normal sailing conditions with a weather helm. The order in which these angles are taken is important to the final attitude. The boat is first rotated about the Z_o axis through the angle ψ. The angle ψ is identical to the boat's leeway angle. The boat is then rotated about the X axis, an angle ϕ, which is the boat's heel angle. The boat is then rotated about the Y_o axis, an angle θ. θ is the angle by which the boat pitches down at the bow and is also the angle of the axis about which the boat heels. Finally the angle α_R is the angle the rudder is deflected from the boat's centerline.

A sailing boat responding to the forces acting upon it and conforming hydrostatically to deformations in the free surface would heave in the positive Z_o direction an amount s.

The selection of these coordinate systems allows the modeling of forces in either coordinate system. When modeling velocity related forces, such as wave drag and lift, the coordinate system fixed in the water is more attractive computationally. On the other hand, hydrostatic forces are more easily computed in the coordinate system fixed in the

boat. In the final solution of force equilibrium, forces and moments are resolved in the X_o, Y_o, and Z_o system except the heeling moment which is resolved about the x axis.

It is a common practice in naval architecture to describe a vessel's dimensions and coefficients in a positive sense. With few exceptions this is done throughout this paper. The positive direction of forces and moments is defined in Section 1.3.2.

1.3.2 Equilibrium of Forces

The forces and moments acting upon a sailing boat in these coordinate systems may be divided into six components:

X = Forces in the positive X_o direction

Y = Forces in the positive Y_o direction

Z = Forces in the positive Z_o direction

K = Moments about the x axis

M = Moments about the y axis

N = Moments about the z_o axis

The positive direction of the moments K, M, and N are taken in the same sense as the angles ϕ, θ, ψ respectively as indicated in Figure 2. Under normal conditions sail aerodynamic forces will create positive forces and moments on the boat.

Through out the remainder of this paper the moments K, M, and N may be referred to interchangeably as forces or moments.

The resolution of forces into equilibrium requires that the summation of all the forces acting in the six directions be equal to zero, i.e.;

$$\Sigma X_i = \Sigma Y_i = \Sigma Z_i = \Sigma K_i = \Sigma M_i = \Sigma N_i = 0$$

where "i" designates individual aerodynamic or hydrodynamic forces or moments acting in the prescribed direction. An example of a contributing force is X_s, the sail driving force. This is one of a number of forces acting along the X_o axis.

The achievement of an equilibrium of forces in the six directions requires the solution of six non-linear simultaneous equations. To obtain a unique solution, six variables are required. A selection of variables for the six degrees of freedom problem is:

$$(U_b, \psi, s, \phi, \theta, \alpha_r)$$

The only unidentified variable is U_b, which is the boat's velocity in the positive X_o direction. Each variable contributes a major effect to a corresponding force. The boat's resistance is strongly dependent on boat speed, U_b. As leeway angle, ψ, strongly contributes to side force, so does heel angle, ϕ, to the righting moment and rudder angle, α_r, to the yaw balance.

The present model excludes the solution in heave and pitch. This exclusion of two degrees of freedom was not due to their lack of importance, but the lack of an adequate parametric model. In reality the axis about which the boat heels may be pitched as severely as 15°. This pitch makes a measurable change in the boat's angle of incidence when heeled to large angles. Further, due to the drop in dynamic pressure as the water passes around the hull a certain amount of sinkage will occur as a measure of the hydrostatic reaction. Besides the effect on form drag, this dynamic pressure and resulting sinkage also increases the boat's wetted surface and viscous resistance. The magnitude of these effects is small enough that they may be reasonably ignored. In the generation of mathematical models for the remaining forces, emperical data was used that came from experimental tests which included the effects of both pitch and heave.

The present model fixes the pitch angle, θ, and sinkage, s, at zero. The variables are reduced to the following four:

$$(U_b, \psi, \phi, \alpha_r)$$

The remaining major forces acting on a sailing boat are resolved into X, Y, K, and N components and are added together to give the following force equilibriums:

$$\Sigma X_i = X_s + X_h + X_k + X_r = 0 \qquad 1$$

$$\Sigma Y_i = Y_s + Y_k + Y_r = 0 \qquad 2$$

$$\Sigma K_i = K_s + K_h + K_k + K_r = 0 \qquad 3$$

$$\Sigma N_i = N_{sx} + N_{sy} + N_k + N_r = 0 \qquad 4$$

The individual forces are described as follows:

X_s = Sail driving force

X_h = Hull resistance

X_k = Hull and keel induced drag

X_r = Rudder induced drag

Y_s = Sail side force

Y_k = Hull and keel side force

Y_r = Rudder side force

K_s = Sail heeling moment

K_h = Hull righting moment

K_k = Keel heeling moment

K_r = Rudder heeling moment

N_{sx} = Yawing moment from sail driving force

N_{sy} = Yawing moment from sail side force

N_k = Yawing moment from keel side force

N_r = Yawing moment from rudder side force.

Each individual force is defined in later sections as a function of the boat's geometry, a true wind vector and the four variables U_b, ψ, ϕ, and α_r. The solution of equations 1 through 4 for a given boat geometry and wind condition provides values for boat speed and attitude. From these values the hydrodynamic performance of the specific sailing boat may then be evaluated.

II. FORCE AND MOMENT MODELS

2.1 Hydrodynamic Forces

The hydrodynamic forces acting on a sailing boat may be conveniently divided into four areas for modeling. The four areas are hull forces, keel forces, rudder forces, and hydrostatic forces. Hull forces consist of resistance from wave making, viscous effects, and form drag. Keel forces are made up of lift on the keel and hull, and drag induced by this lift. Rudder forces in the same way consist of lift and induced drag from the rudder. Finally, hydrostatic forces are caused by the change in position of the center of buoyancy in the hull. Each of these areas is presented individually and models are developed for their contribution to the total force equilibrium.

2.1.1 Hull Resistance

The estimation of hull resistance is a problem faced by naval architects in the design of every ocean going vehicle. The difference, in predicting a sailing boat's resistance, is in the determination of the effects changes in attitude have on these calculations. The approach taken is to determine a model for up-right resistance and modify this resistance as necessary to account for changes in the boat's attitude.

Upright Resistance

The technique commonly used for prediction of resistance from tow tank data is based on Froude's Hypothesis. Basically, the hypothesis states that the total upright resistance may be divided into two components. The first is frictional or viscous resistance. Frictional resistance is dependent on Reynold's Number, a non-dimensional parameter defined as:

$$R_n = U_b L/\gamma_{H_2O}$$

The second component is residual resistance. The residual resistance is the combination of drag due to the generation of waves and the effects of eddy making and form drag. The effect of the hull generating waves is a gravitational effect, largely dependent on Froude Number. This non-dimensional coefficient is defined as:

$$F_n = U_b/\sqrt{gL}$$

Though Froude left the separation of the elements of resistance in these two components, form drag and eddy making resistance have little dependence on either Froude or Reynolds Numbers over the velocity ranges encountered by sailing boats. For this reason it is logical to further separate residual resistance into wave resistance and form resistance.

The separate components of upright resistance are assumed independent of each other so that the total resistance of a boat at a given speed may be estimated by superposition of the forces. Total upright resistance is then:

Upright resistance = frictional resistance

+ wave resistance + form resistance.

Each component of upright resistance is examined in detail and mathematical expressions are developed to quantify their magnitude.

Wave Resistance

The mathematical model for wave resistance is based on a generalized form of Michell's Integral. This expression is evaluated for a base hull form and is corrected for parametric deviation from this parent form. This expression is emperically adjusted to give accurate prediction of the towing tank results of Antiope (3).

Michell (7) postulated that wave resistance for thin ships depends upon the computation of pressure changes, δ_p, due to the wave pattern. His expression for this resistance is

$$R_w = \frac{4\rho g}{\pi U_b^2} \int_1^\infty (I^2+J^2) \frac{\lambda}{\sqrt{\lambda^2-1}} d\lambda \qquad 5$$

where

$$I = \int_{-e}^{+e} \int_{0}^{K(x)} \frac{\partial y}{\partial x} \; e^{-\lambda^2 gz/U_b^2} \; \cos\left(\frac{\lambda g x}{U_b^2}\right) \, dx \, dz \qquad 6$$

$$J = \int_{-e}^{e} \int_{0}^{K(x)} \frac{\partial y}{\partial x} \; e^{-\lambda^2 gz/U_b^2} \; \sin\left(\frac{\lambda g x}{U_b^2}\right) \, dx \, dz \qquad 7$$

Another form of these equations was derived by Weiblum (8). A general expression of his result is

$$R_w = \rho g \frac{B^2 H^2}{L} \, F\left(\frac{L}{T_c}, \text{ form coefficients}, F_n\right) \qquad 8$$

By assuming that the block coefficient, C_b, varies very little between yacht forms, equation 8 may be written as:

$$R_w = \frac{L^3}{\rho g} \left[\frac{\Delta}{(L/100)^3}\right]^2 \, f\left(\frac{L}{T_c}, \text{ form coefficients}, F_n\right) \qquad 9$$

The number of experiments conducted to validate Michell's result is small. Wigley (9) compared model results to theory and observed the interference humps in the resistance curve were overpredicted by theory. Further comparison between vessels of constant draft and varying beam done by Wigley (10), Weinblum (11), and Mumford (12) indicate for slender ships wave resistance is approximately proportional to the square of the beam as estimated by Michell's Integral, but as the beam to length increases dependence of resistance on beam appears to be less than quadratic. It is expected that the

high beam to length ratios encountered in sailing boats will strain the slender ship limits of Michell's Integral.

The basic form of the wave resistance is rewritten as:

$$R_w = \frac{L^3}{\rho g} \left[\frac{\Delta}{(L/100)^3}\right]^n f(L/T_c, \text{ form coefficients, } F_n) \qquad 10$$

where n is between one and two. The actual value is determined by emperical data discussed in the following sections.

The identification of the most important parameters affecting wave resistance is required. Todd (13) did a regression analysis of over two hundred destroyers and found the following coefficients had the highest correlation between the speed length ratios of 0.8 and 1.25.

$$(\nabla/(L/100)^3, C_p, LCB, L/T_c, F_n)$$

For geometrically similar boats we know directly from Froude's Hypothesis that the wave resistance scales is the cube of the characteristic length. The general form of Michell's Integral, equation 10, is used for the dependence on displacement to length ratio. By assuming base value for prismatic coefficient, C_{po}; longitudinal position of the center of buoyancy, LCB_o; and length to draft ratio, L/T_{co}; an expression for the wave resistance of a parent form may be written:

$$R_{wp} = f_1(U_b/\sqrt{L}) \frac{L^3}{\rho g} \left[\frac{\Delta}{(L/100)^3}\right]^n \qquad 11$$

at C_{po}, LCB_o, and L/T_{co}.

To predict the wave resistance of a sailing boat, the resistance of the parent boat is adjusted by correction factors based on the boat's geometric deviation from the parent. The wave resistance equation is

$$R_w = R_{wp} \text{ (1 + } C_p \text{ correction factor + LCB correction factor + } L/T_c \text{ correction factor)} \qquad 12$$

Each component of this equation is defined and discussed in the following sections.

Characteristic Length - L

The characteristic hydrodynamic length or "effective length" of a sailing boat is difficult to quantify. This has been a difficulty in the maintenance of all offshore rating rules as demonstrated by the extremes to which designers have gone to make rated length look shorter than the boat's actual hydrodynamic length. For the purposes of this model, L is determined to be the load water line, LWL. This creates little difficulty when examining the comparitive performance of similar forms, but subjective adjustment would be required

when comparing plumb bow, transom stern boats to long ended boats with shallow buttocks and entrance profiles. To some extent the latter might increase in length with Froude Number as the boat will be supported more by her overhangs.

Parent Froude Number Dependence - $f_1(U_b/\sqrt{L})$

The speed length ratio, $U_b/\sqrt{L}$, was adopted instead of the dimensionally correct variable $F_n = U_b/\sqrt{gL}$ due to the common usage and the comfortable feel most yacht designers have with speed length ratio. The function, $f_1(U_b/\sqrt{L})$ adjusts the wave resistance equation to accurately predict the resistance of the yacht Antiope. The function was determined by inputting Antiope's geometry into the model and selecting $f_1(U_b/\sqrt{L})$ to insure proper correlation with the full scale towing test results reported in (3) and (14). Values of the function were established at each data point given in (14) and a spline cubic curve was fitted to give the curve in Figure 3 for n = 2.0.

Displacement to Length Factor - $[\Delta/(L/100)^3]^n$

The high beam to length ratios, B/L, encountered in sailing boats requires that the exponent, n, be determined from emperical data. Unfortunately, the data base from tests of actual sailing boats is small. The scale effects pointed out in reference (15) precludes the use of results from towing tests of small scale yacht forms.

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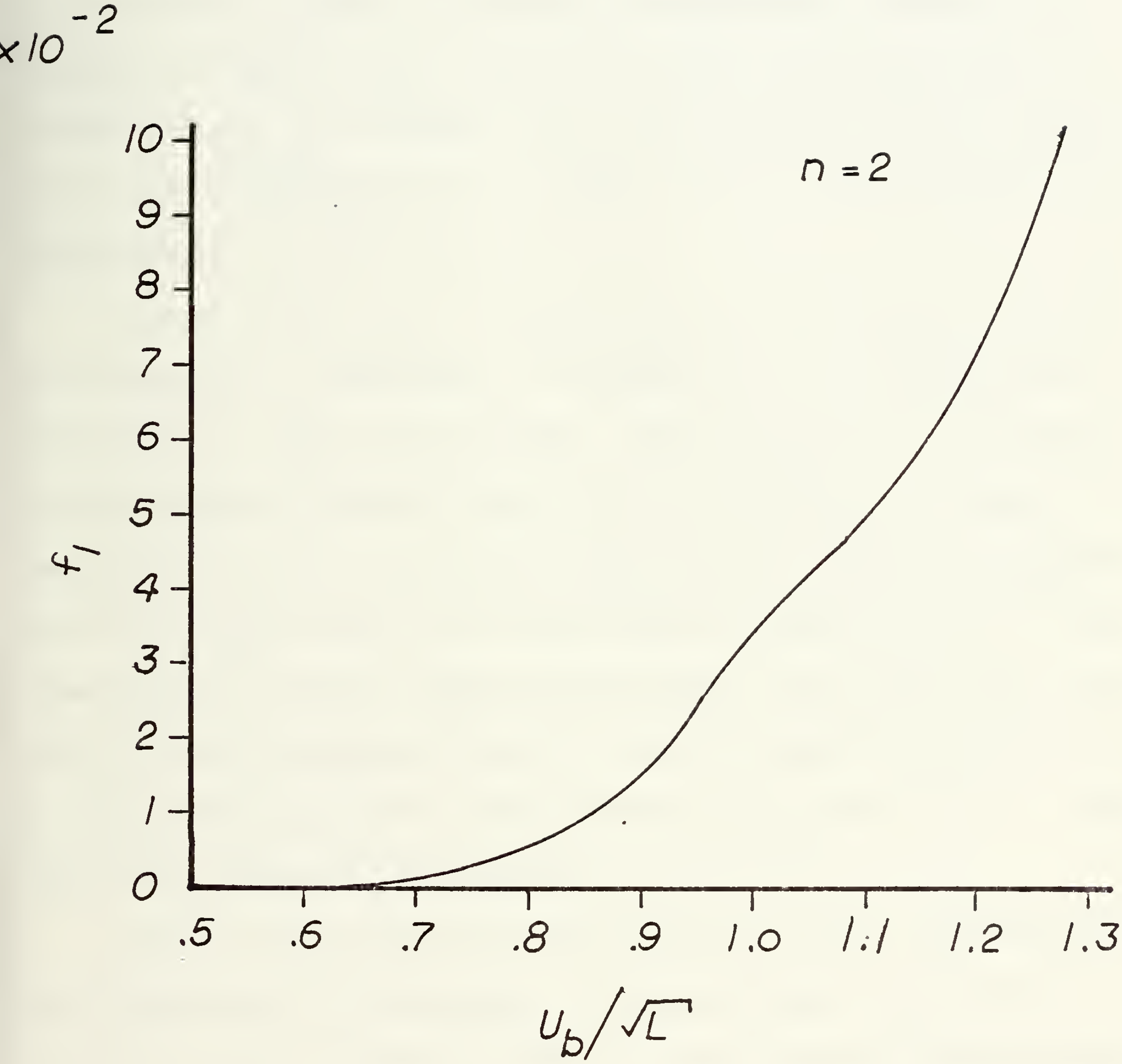

Figure 3

Parent Froude Number Dependence

Examination of Taylor Standard Series (16) at prismatic coefficients typical in yacht design indicate that a value of n between 1.8 and 2.0 appears reasonable. Though the prismatic coefficients, length to draft and displacement to length ratios in reference (16) are typical of sailing boats, the block coefficient, C_b, is high and not characteristic of yacht forms.

Figure 4 is a plot of two curves predicting the wave resistance of a Standfast 43 sailboat by the mathematical model with n equal one and two. Also plotted are the data points from the towing tests of a one sixth scale model conducted by M.I.T. and Delft University. Figure 5 compares the full scale test (15) of the one ton yacht Bullet to two theoretical curves. The form drag data from both yachts was taken as the residuary drag at a speed length ratio equal to 0.4 and subtracted from the residuary resistance versus speed curve to obtain the wave drag.

Two is maintained as the value of n in the computerized model appended to this paper. Though comparisons made in Figures 4 and 5 indicate that n = 2 may slightly overestimate the effect displacement to length ratio has on wave making resistance, the limited sample size available for correlation does not justify adjustment from the theoretical value.

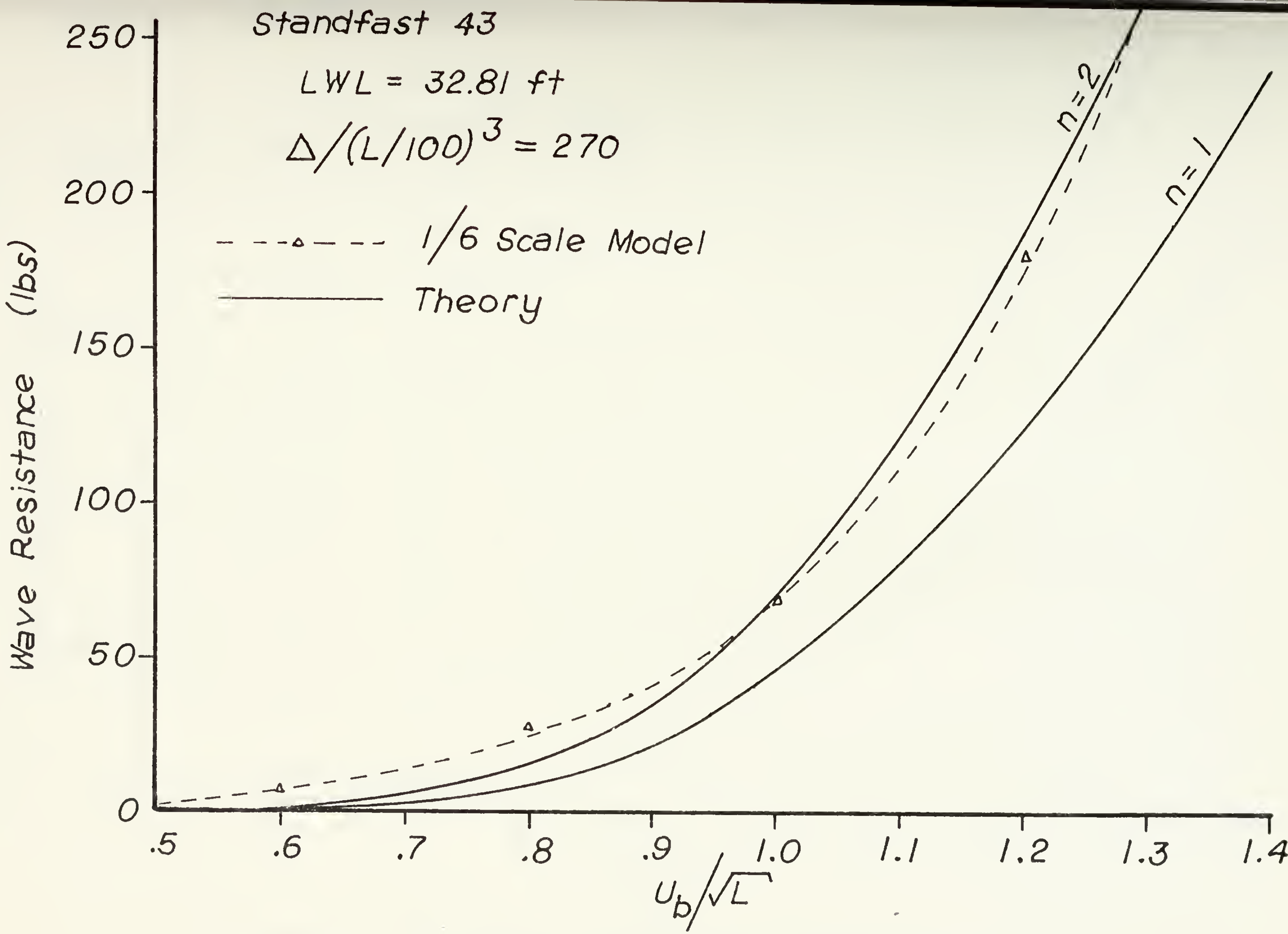

Figure 4 - Standfast 43 Wave Resistance

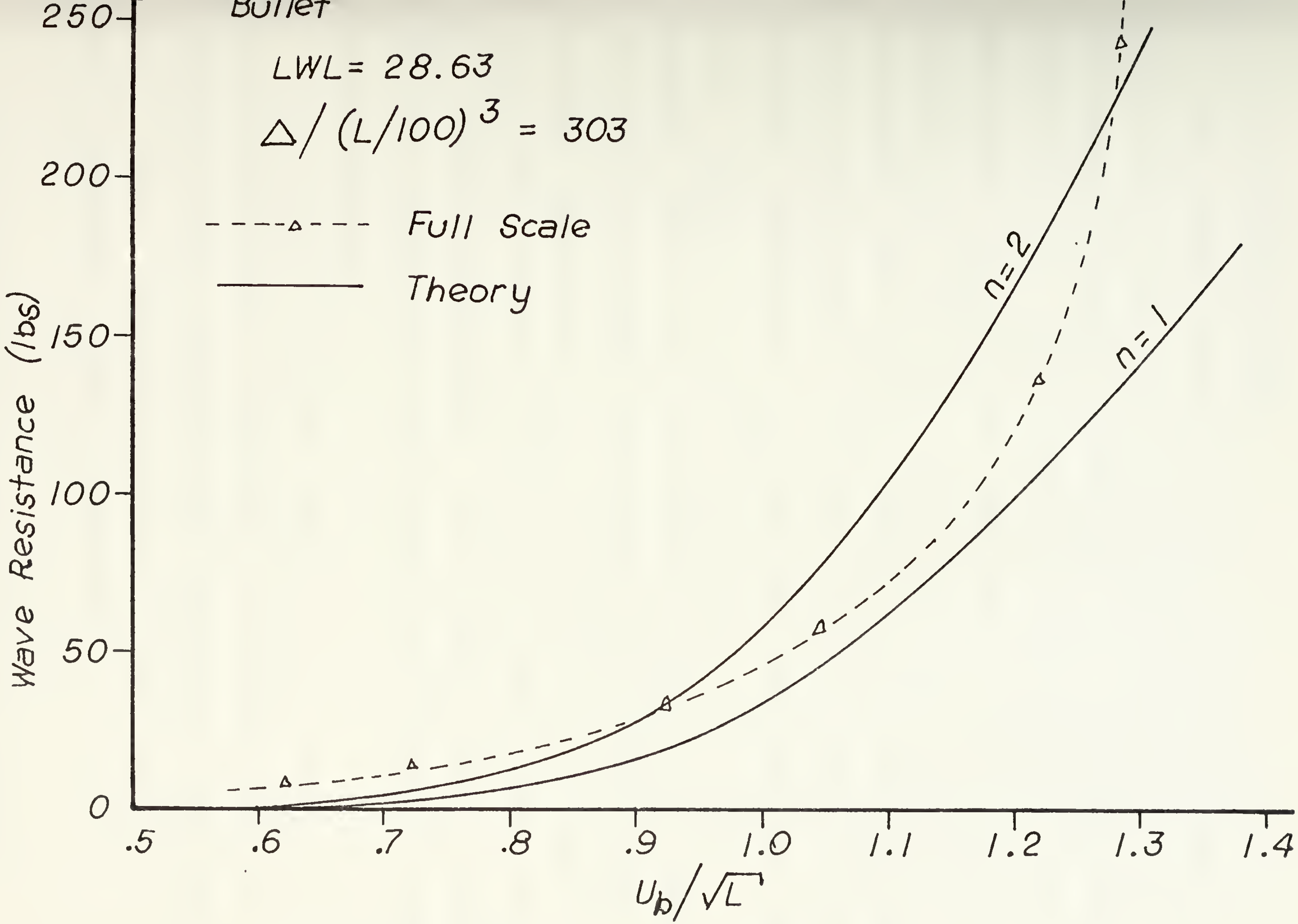

Figure 5 ~ Bullet Wave Resistance

C_p Correction Factor

The prismatic correction factor corrects the magnitude of the sailing boat's wave resistance for deviations away from the base prismatic coefficient, C_{po}. The base prismatic coefficient, C_{po}, was selected to be the optimum prismatic coefficient, C_{popt}, for each speed length ratio. Taylor's Standard Series (16) data are the only tests available of large models where prismatic coefficient was systematically varied. Examination of these data reveals a definite optimum prismatic coefficient, C_{popt}, for each tabulated speed length ratio. Figure 6 is a spline cubic fit of this optimum prismatic taken at the displacement to length ratio equal to 250 and plotted versus speed length ratio. This optimum prismatic coefficient, C_{popt}, as a function of the speed length ratio is selected as the base prismatic coefficient, C_{po}, of the parent equation. This is done so that deviations from this optimum will always result in an increase in wave making resistance.

From Taylor's Standard Series, reference (16), the percentage increase in resistance for deviation from the optimum prismatic coefficient was tabulated for each speed length ratio. From this emperical data the expression for the C_p correction factor is generated.

$$C_p \text{ correction factor} = f_2(U_b/\sqrt{L})\,|100(C_{popt}-C_p)|^{1.7} \qquad 13$$

Figure 7 is a plot of the spline cubic fit of the correction factor's dependence on speed length ratio, $f_2(U_b/\sqrt{L})$. A reasonable definition for the sailing boat's prismatic coefficient, C_p, must be established to insure quantitative agreement between the Taylor Series, reference (16), forms and the yacht forms. A method for weighting the volume and sectional area of the keel in the calculation of the longitudinal prismatic coefficient is developed. Equations 6 and 7 of Michell's Integral indicate that displacement should be weighted exponentially in the vertical direction by

$$e^{-\lambda^2 gz/U_b^2}.$$

There are two common practices in calculating the C_p of a sailing boat. The first is to include the total volume and sectional area of the keel into the calculation. The inclusion of all the deeply placed sectional area of the keel would overestimate its effect on the wave making resistance when compared to the keel's actual effect on the wave train. On the other hand, the second method is to treat the keel as an appendage and calculate only a fairbody or canoe prismatic. This ignores the keel's displacement effect altogether. Typical values of C_p for the same yacht calculated both ways are 0.52 and 0.58 with and without the keel included respectively. If the Taylor Series, reference (16), optimum is to be used, the corresponding prismatic coefficient would be somewhere in between these two values.

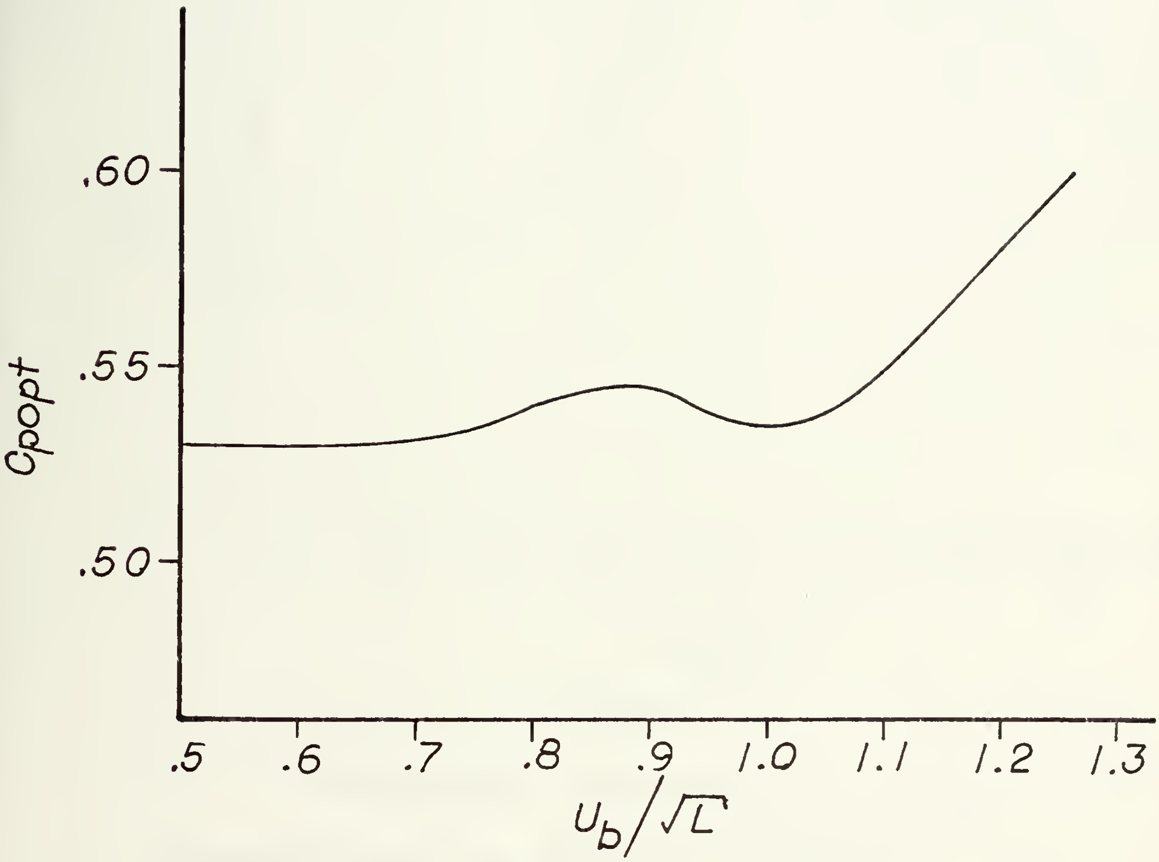

Figure 6 - Optimum Prismatic Coefficient

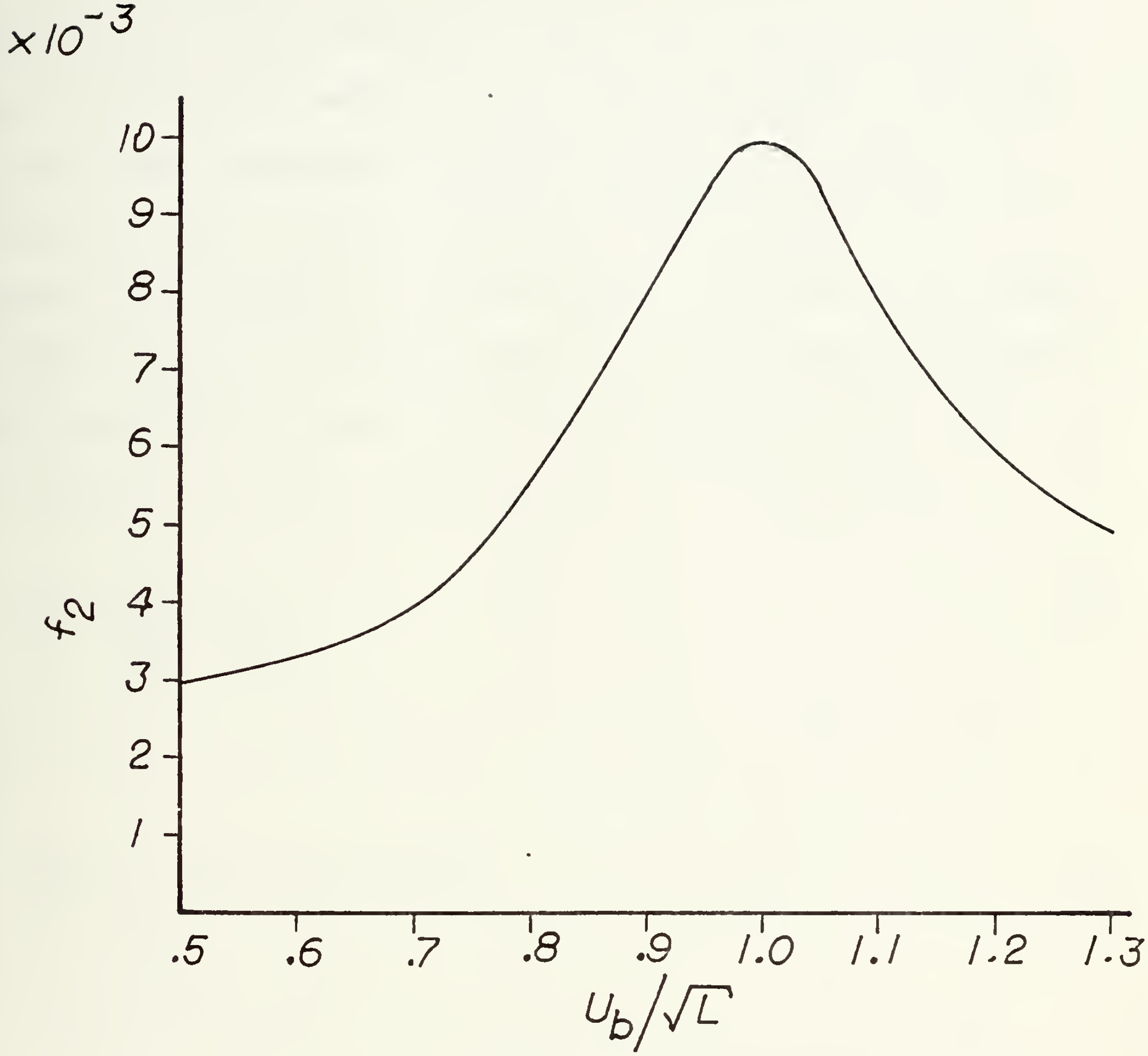

Figure 7

Prismatic Correction Factor

Dependence on Froude Number, f_2

One would expect the volume of the keel to have its largest effect when the sailing boat is sitting on a wave of approximately the same length as the waterline. At this point the wave trough would bring the keel the closest to the free surface. An estimation of keel's proper effect on prismatic coefficient, C_p, may be made by weighting the keel's area and volume by e^{-1/F_{nz}^2} where F_{nz} is the vertical Froude number evaluated at speed length ratio equal to one. The longitudinal prismatic coefficient, C_p, is defined as:

$$C_p = \frac{\nabla_c + \nabla_k^*}{(A_m + A_k^*)\ L} \qquad 14$$

where

$$\nabla_k^* = \nabla_k\ e^{-32.2d/(1.689L)^2} \qquad 15$$

and

$$A_k^* = A_k\ e^{-32.2d/(1.689L)^2} \qquad 16$$

∇_c and ∇_k are the displaced volume of the canoe body and keel respectively. A_m and A_k are the maximum cross sectional area of the canoe body and the cross sectional area of the keel at that station respectively. The depth, d, is measured from the static waterline to the center of the keel's displaced volume.

L/T_c Correction Factor

Taylor Standard Series (16) tabulates data on ships at length to draft ratios equal to eight and sixteen. Taylor Series (16) asserted that linear interpolation between these ratios was adequate.

The base length to draft ratio, L/T_{co}, of 16 is selected from Taylor Series (16) data and the correction factor is

$$L/T_c \text{ correction factor} = -f_3(U_b/\sqrt{L})(16-L/T_c) \qquad 17$$

where $f_3(U_b/\sqrt{L})$ is a spline cubic fit of the speed length effect as shown by Taylor (16). $f_3(U_b/\sqrt{L})$ is plotted in Figure 8.

The major differences between the ships tested for Taylor's Standard Series (16) and yacht forms are in the shape of the sections. The Taylor form typically has a maximum section coefficient, C_m, of 0.98 while sailing boats range between 0.60 and 0.70. As a result, the vertical distribution of volume between the two forms differ to a large extent. It is not completely accurate to assume that vertical shifts in the volumetric distribution of the two forms will have a similar effect on wave making resistance. The model utilizes the best data available to date, but the need exists for a large scale yacht series examining just such a systematic parameter variation.

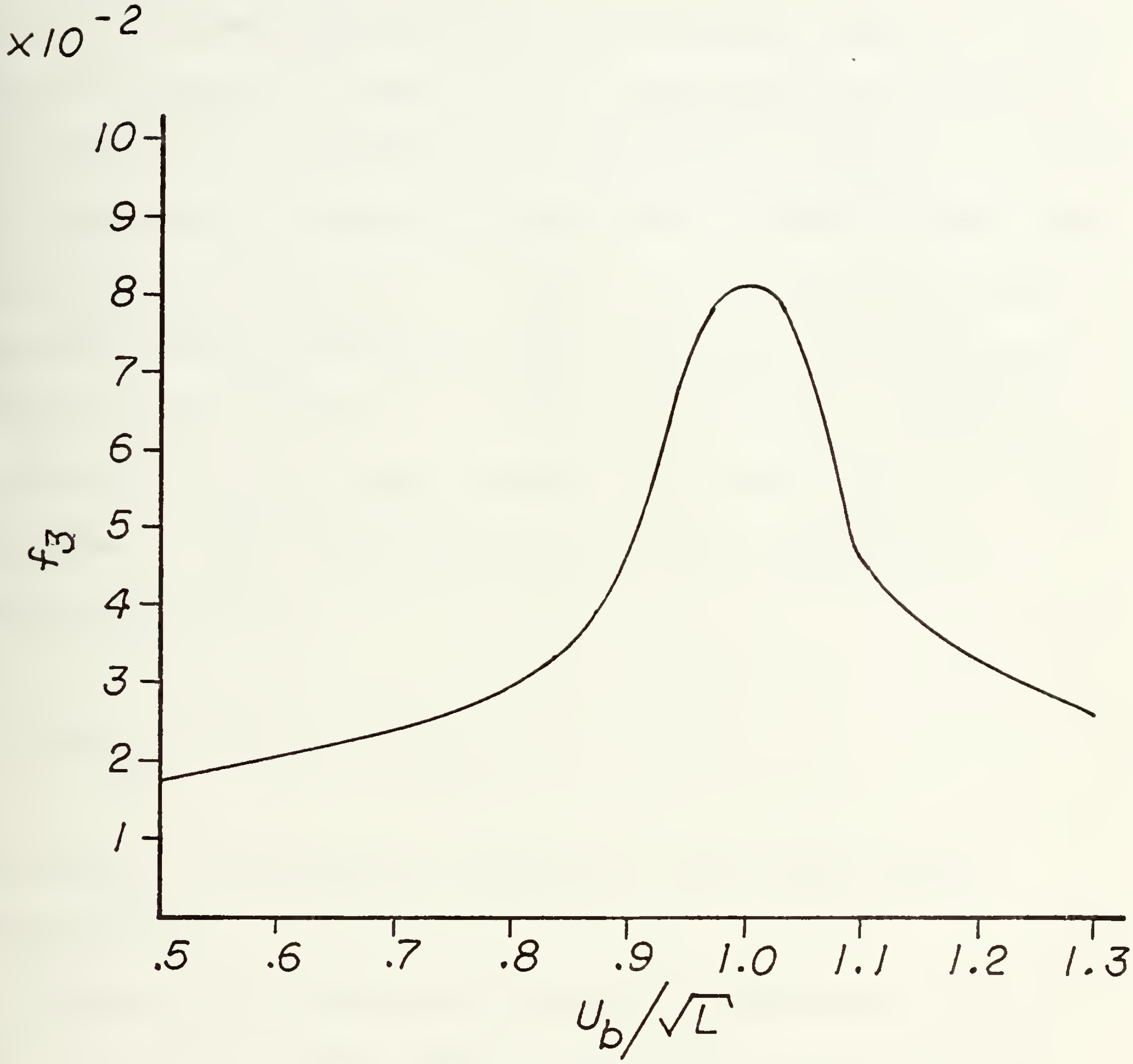

Figure 8

L/T_c Correction Factor

Dependence on Froude Number

LCB Correction Factor

In the same manner as the C_p correction factor, the LCB correction factor is based on the actual LCB and an optimum LCB position. A summation of recommended optimum LCB positions as a function of block coefficient and prismatic coefficient was presented by Todd (17). There is reasonable agreement among the recommended curves with less than 1.5% variation between extreme curves in the area of interest. Comparing the LCB curves with common practice in yacht design a curve attributed to Professor Troost in (17) was adopted. In modified form it is

$$LCB_{opt} = 62.5 - 17.5C_p \qquad 18$$

where LCB_{opt} is measured in percent L from the forward perpendicular.

Examination of the major series in references (17), (19), and (20) which consider LCB as a variable, none adequately cover the region in which sailing boats fall. In all cases, these series cover block coefficients higher than typical yachts. Certain trends may be determined though:

1. As with the previous factors, there is a peak effect at speed length ratio equal to one.
2. As block coefficient decreases, the penalty for non-optimum center of buoyancy position also decreases.

The above series, therefore, place an upper bound on the expected penalty.

Todd (17) indicates for Series 60 that a one percent change from the optimum LCB position will increase wave making resistance by approximately one percent. Beyond a one percent change the penalty increases sharply.

The assumed factor is

$$\text{LCB correction factor} = \sin(90U_b/\sqrt{L})\,|62.5-17.5C_p-\text{LCB}|^3/100$$

19

Frictional Resistance

The mathematical model for frictional resistance is derived from procedures adopted by the Eighth International Towing Tank Conference (21), ITTC in 1957. Traditionally frictional or viscous resistance is defined as:

$$R_f = C_f\, 1/2\rho\ S_w\ U_b^2$$

S_w is the wetted surface of the sailing boat and C_f is the coefficient of frictional resistance.

The ITTC defined the coefficient of frictional resistance as:

$$C_f = 0.075/(\log_{10} R_n - 2)^2$$

where Reynold's Number, R_n, is the only parameter affecting this coefficient. Again, Reynold's Number is defined as:

$$R_n = U_b L/\gamma_{H_2O}$$

where γ_{H_2O} is the kinematic viscosity of water.

A suitable definition of the Reynold's Number characteristic length, L, must be established. The ITTC adopted the use of the load waterline length, LWL, as the characteristic length. This makes sense for ships with rectangular profiles, but sailing boats characteristically have cut away profiles and keel and rudder chords which have little dependence on the length of the waterline. Davidson (22) realized this problem and adopted seventy percent of the waterline length as the characteristic length. The adoption of a constant length lacks sensitivity to the effects differing profiles have on the characteristic length. To make the mathematical model sensitive to these variations, the total wetted surface of the yacht is divided into three areas, each with its own characteristic length.

The three areas are the sailing boat's canoe body, the keel and the rudder. The load waterline length, LWL, is the characteristic length of the canoe body while the mean chord of the keel, $\bar{c}_k$, and rudder, $\bar{c}_r$, are the characteristic lengths of the keel and rudder respectively.

The coefficient of friction for each area is calculated using equation 21. The total frictional resistance is the sum of the resistances of each area and is written as:

$$R_f = 1/2\rho_{H_2O}\, U_b^2\, (C_{ffb}S_{fb}+C_{fk}S_k+C_{fr}S_r)$$

The above coefficients and wetted surfaces are defined as:

C_{ffb} = fairbody coefficient of friction

S_{fb} = fairbody wetted surface

C_{fk} = keel coefficient of friction

S_k = keel wetted surface

C_{fr} = rudder coefficient of friction

S_r = rudder wetted surface

The ITTC equation for the coefficient of friction is an emperical formulation which historically has either slightly overpredicted or underpredicted frictional resistance for certain forms. From previous experience with a given form, a factor may be added to give satisfactory correlation with full scale results. When equation 22 is applied to Antiope (3), the correlation factor is unnecessary. This would indicate that the yacht Antiope has no form resistance or skin roughness. This is unfounded and indicates that the ITTC formulation overpredicts the frictional resistance of the yacht Antiope by the approximate magnitude of its own form resistance. This effect is discussed further in the next section.

Form Resistance

Form resistance is a combination of effects which show little dependence on Froude Number and Reynold's Number in the speed range seen by sailing boats. Form resistance, R_{fm}, is defined as:·

$$R_{fm} = C_{fm}\ 1/2\rho S_{fb} U_b^2 \qquad 24$$

The form resistance coefficient, C_{fm}, is a constant with speed and depends only on the boat's geometry, surface roughness, and appendages. Form resistance is made up of eddy making resistance, roughness effects, and pressure drag due to separation.

The form resistance of a yacht may constitute ten percent of the boat's total resistance at low speed and decreases to less than two percent as the effects of wave making resistance increase. Kirkman and Pedrick (15) discuss the difficulty in modeling form resistance from analytic and emperical sources. The scale effects from small scale model tests have made the measurement of this component difficult. Further, the selection of the ITTC friction coefficient formulation includes an arbitrary amount of form resistance which is not quantifiable. The use of this line in predicting Antiope (3) resistance did not require the use of a form drag coefficient to provide correlation.

The present mathematical model does not parameterize the form resistance coefficient but allows the user to input a value to improve correlation with known results. The inclusion of this coefficient allows future parameterization as techniques for its prediction become available.

Hull Force Formulation

The total upright resistance, R_u, is the superposition of wave resistance, frictional resistance, and form resistance as defined by equations 12, 23, and 24.

$$R_u = R_w + R_f + R_{fm} \qquad 25$$

The change in hull resistance with heel and leeway angle must be considered. For small angles of leeway, ψ, the change in resistance is considered an effect of lift and is modeled in the section on keel forces, 2.1.2.

In the absence of lift, the change in upright resistance due to heel, ϕ, is caused by a number of effects. As the sailing boat heels, one would expect not only changes in the boat's symetrical lines, but also her form. Depending on the yacht form, the effective sailing length may also increase or decrease. Further, as the boat heels, the keel moves closer to the free surface and in effect decreases the effective prismatic coefficient and increases resistance.

Examination of a number of towing tank tests conducted at M.I.T. show that the increase in resistance at zero side force varies depending on the design. This variation ranged from an increase in resistance from a minimum of five percent to a maximum of twenty-five percent at twenty-five degrees heel. The majority of the well-designed models show an increase of approximately ten percent at twenty-five degrees heel. Because of the lack of test date available which systematically investigates heel effects, the increase in resistance with heel is limited to an estimate of what good design practice has demonstrated. This effect is emperically modeled by dividing the upright resistance by the cosine of the heel angle.

The expression for the hull forces in the X_o direction is written as:

$$X_h = R_u / \cos \phi \qquad 26$$

where R_u is the upright resistance defined by equation 25.

2.1.2 Keel Forces

The mathematical models for lift forces acting on the hull and keel are derived from slender body theory as applied to fish-like bodies in references 23, 24, and 25. Expressions are defined for the hull and keel side force, Y_k; induced drag, X_k; yawing moment, N_k; and the heeling moment, K_k.

Hull and Keel Side Force

Letcher (3) investigated the possibility of utilizing available theories for predicting a sailing boat's side force. He compared lifting line theory, slender body theory and an adoption of slender-body theory for fish-like bodies to the towing tank test results of the yacht Antiope. The theory presented by Newman and Wu (24) for fish-like bodies demonstrated excellent correlation to Letcher's (3) regression analysis of Antiope's test data. An adaptation of this theory is utilized for modeling the hydrodynamic lifting forces acting on the keel and hull.

The side force acting on the keel and hull is defined as:

$$Y_k = -C_{y\psi}\cos^2\phi\,(1/2\rho_{H_2O}\;L^2U_b^2\psi) \qquad 27$$

where the $\cos^2\phi$ term was found by Letcher (3) to satisfactorily quantify the effect heel has on the side force coefficient, C_y, for the yacht Antiope. L is the load waterline length.

A mathematical model for side force coefficient is derived from slender-body theory as applied to fish-like forms. Milgram (23) gives an expression for the local lift, L(x), on a slender-body as:

$$L(x) = \frac{\pi}{2}\rho_{H_2O}\,\frac{d}{dx}\,[b^2(x)]V_bU_b \qquad x_o < x < x_b \qquad 28$$

where b(x) is the local span, x_o is the horizontal position of the maximum span, and x_b is the horizontal distance from the maximum span to the forward perpendicular. Figure 9 shows Antiope's profile with the locations of x_o and x_b. V_b is the component of velocity y direction.

The total side force predicted by slender-body theory is

$$Y_{sb} = \int_{x_o}^{x_b} L(x)\, dx \qquad 29$$

For the numerical computation of the slender-body side force, the yacht's profile is tabulated at the five points shown in Figure 9 and the profile, b(x), is approximated by straight lines between these points. Equation 29 is evaluated numerically to give the slender-body lift on the hull and keel.

The velocity V_b may be approximated for small leeway angle by the linear expression:

$$V_b = U_b\psi \qquad 30$$

The coefficient of side force, $C_{y\psi sb}$, based on slender-body theory may be written as:

$$C_{y\psi sb} = Y_{sb}/\tfrac{1}{2}\rho_{H_2O}\, L^2\, U_b^2\psi \qquad 31$$

where L is taken as the waterline length, LWL.

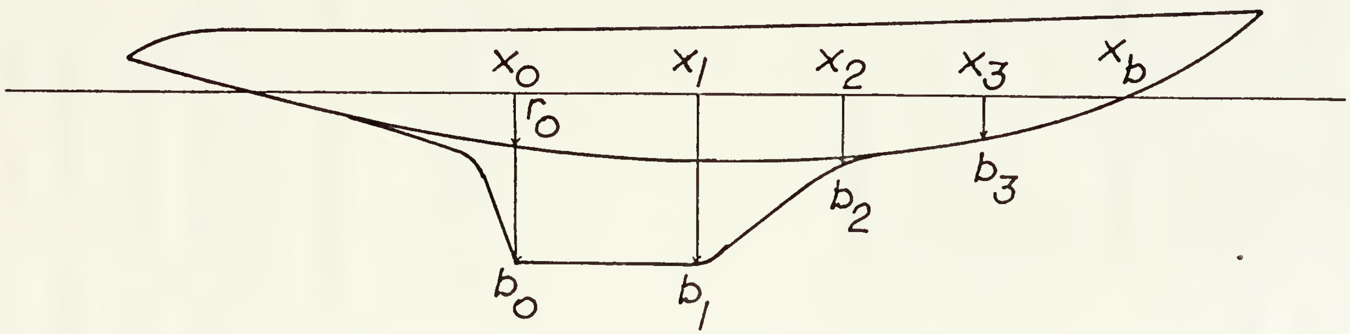

Figure 9 - Antiope Profile

Newman and Wu (24) developed a procedure for determining the effect an axisymetric body of radius r_o has on the slender-body lift. From reference 24, Figure 10 is a plot of $-L/\pi\rho U_b V_b b_o^2$ versus r_o/b_o. At r_o/b_o equal to zero, the curve gives the slender-body lift for a given span. As r_o/b_o increases the lift is attenuated by the amount shown in the curve. This curve may be defined as a fair body factor, f_{fb}. The curve is entered with the ratio r_o/b_o to obtain the fair body factor. The slender-body coefficient from equation 31 is multiplied by the fair body factor to compute the yacht's side force coefficient. The expression for the side force coefficient is

$$C_{y\psi} = C_{y\psi sb}(f_{fb}) \qquad 32$$

The radius, r_o, is measured as the draft of the canoe body at the maximum draft measurement, b_o. These two measurements are depicted in Figure 9.

This technique for the solution of side force correlates well with the full scale tests of the yacht Antiope (3). Antiope has a moderately long keel compared to modern sailboats with their fin keels. For a given hydrodynamic span, as the chord of a long keel decreases, the lift remains essentially unchanged, but at a certain point a further decrease in the keel's chord can no longer support the spanwise elliptic

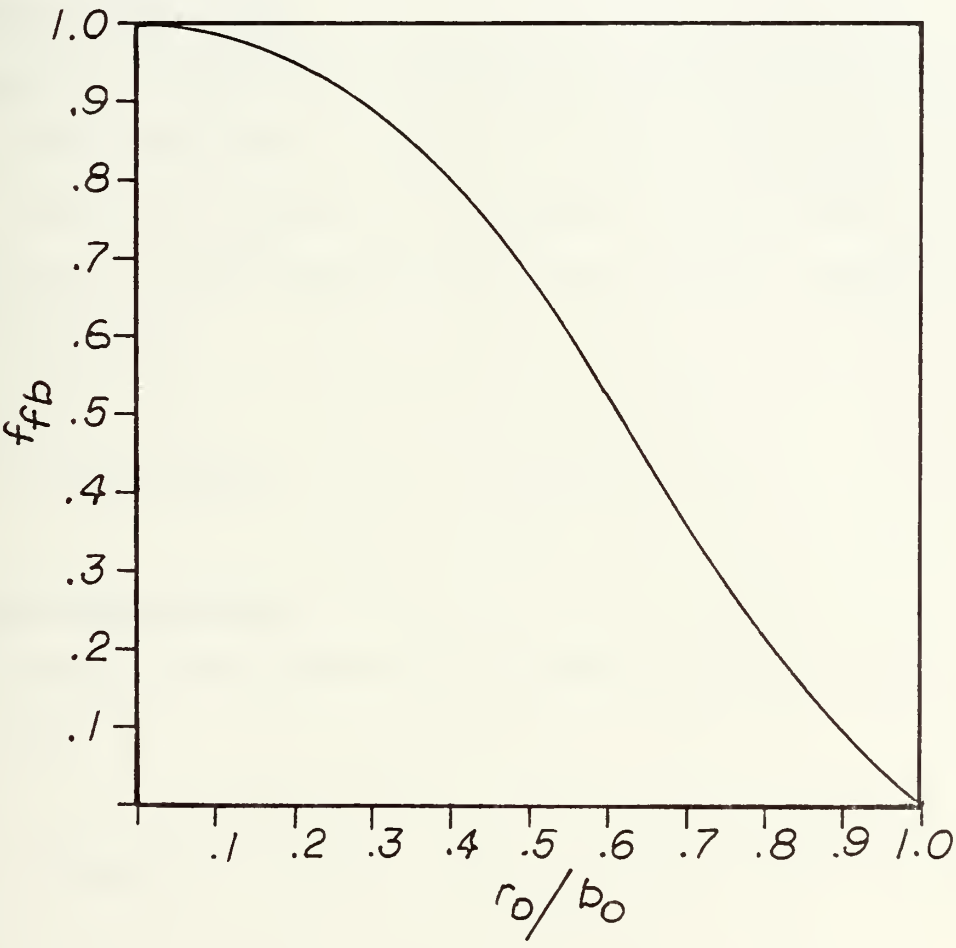

Figure 10 - Lift Attenuation

loading assumed by slender-body theory. The present model is not sensitive to this effect, and though the model is adequate for modeling changes in draft, it should not be utilized to optimize keel area.

Keel Induced Drag

Newman in reference 25 develops the equation for induced drag on fish-like forms. Figure 11 is from reference 25 and gives a curve for $(\psi Y_k/D_i)$ as a function of the ratio r_o/b_o. By generating a spline cubic fit to this curve, induced drag, X_k, is defined as:

$$X_k = Y_k\psi/(\psi Y_k/D_i) \qquad 33$$

Keel Yawing Moment

The yawing moment due to the side force on the keel is defined as:

$$N_k = x_{cpk}Y_k \qquad 34$$

The center of effort of the hydrodynamic side force, x_{cpk}, is determined from slender-body theory. Milgram (23) gives the yawing moment about x_o in Figure 9 as:

$$N_{sb} = \int_{x_o}^{x_b} xL(x)\,dx \qquad 35$$

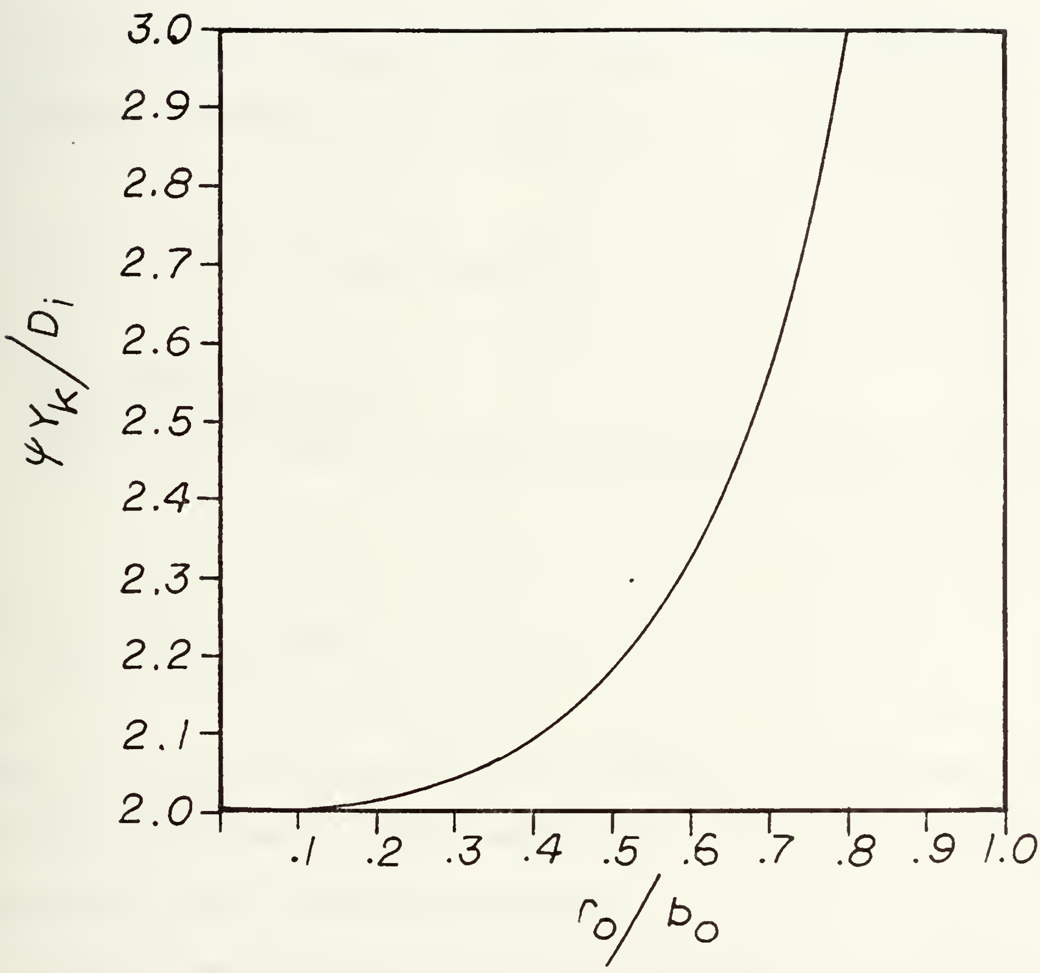

Figure 11 - Induced Drag

where L(x) is defined by equation 28. Hence, the non-dimensional coefficient for yawing moment can be written as:

$$C_{n\psi sb} = N_{sb}/\tfrac{1}{2}\rho H_2O\ L^3\ U_b^2\psi \qquad 36$$

and the distance to the center of hydrodynamic effort from the forward perpendicular, x_{cpk}, as:

$$x_{cpk} = (1.0 - C_{n\psi sb}/C_{y\psi sb})L \qquad 37$$

<u>Keel Heeling Moment</u>

The heeling moment caused by the side force on the keel is

$$K_k = -Y_k z_{cpk} \cos\psi \qquad 38$$

where z_{cpk} is the distance to the vertical hydrodynamic center of effort measured from the waterline. Since the leeway angle, ψ, is small, $\cos\psi$ is taken as unity.

Slender-body theory tends to overpredict the location of the vertical center of effort when compared to towing tank data. Letcher (3) reduced Antiope data for the location of the center of effort and deduced that the vertical center of effort was approximately 1.5 feet below the waterline. From this a reasonable estimate for the vertical center of effort, z_{cpk}, is made.

$$z_{cpk} = 0.33b_o \tag{39}$$

where b_o is the maximum draft as defined in Figure 9.

2.1.3 Rudder Forces

Mathematical models for the forces acting on the rudder are derived from the emperical equation for spade rudders presented in reference (26). Expressions are defined for the rudder side force, Y_r; rudder drag, X_r; rudder yawing moment, N_r; and rudder heeling moment, K_r.

Rudder Side Force

Whicker and Fehlmer (26) conducted a series of wind tunnel tests on low and medium aspect ratio spade rudders and provided emperical equations which accurately model the forces and moments acting on a large family of spade rudders. The equations are sensitive to aspect ratio, a_r, sweep angle, Ω, and taper ratio, t_r.

The modeling of rudder forces in relation to a sailing boat are complicated by the effect of the downwash from the keel. By treating the keel as a low aspect ratio lifting surface, the downwash can be assumed to deflect the flow along the centerline of the boat. Although the keel will actually deflect the flow slightly less than this, the approximation is considered reasonable. The model therefore assumes that the angle of attack on the rudder and the rudder's deflection from the boat's centerline, α_r, are identical. The effect of heel on rudder performance is considered negligable.

Figure 12 is a drawing of a spade rudder including the measurements and nomenclature used in determining the rudder's performance. The geometrical ratios required by the theory are derived from these measurements. The hydrodynamic aspect ratio, a_r, is defined as:

$$a_r = 2b_r/\bar{c}_r \qquad 40$$

where b_r is the geometric span and $\bar{c}_r$ is the mean chord as defined in Figure 12. Taper ratio, t_r, is defined as:

$$t_r = c_{rt}/c_{rr} \qquad 41$$

where c_{rt} is the rudder tip chord, and c_{rr} is the rudder root chord as defined in Figure 12.

Whicker (26) defines the coefficient of lift as:

$$C_{1r} = \left(\frac{\partial C_L}{\partial \alpha_r}\right)\alpha_r + \frac{C_{dc}}{a_r}\left(\frac{\alpha_r}{51.3}\right)^2 \qquad 42$$

where C_{dc} is the cross flow drag coefficient, and the rudder angle, α_r is measured in degrees.

An emperical expression from reference (27) for the cross flow drag coefficient, C_{dc}, for rudders with faired tips is

$$C_{dc} = 0.08 + 0.72t_r \qquad 43$$

direction to starboard. The z axis desends orthogonally to the x-y plane in the direction of the keel.

The angular differences between the two coordinate systems describe the attitude of the boat as it moves through the water. The positive direction for these angles are also presented in Figure 2. Their direction was selected to provide positive values when the boat is in equilibrium under normal sailing conditions with a weather helm. The order in which these angles are taken is important to the final attitude. The boat is first rotated about the Z_o axis through the angle ψ. The angle ψ is identical to the boat's leeway angle. The boat is then rotated about the X axis, an angle ϕ, which is the boat's heel angle. The boat is then rotated about the Y_o axis, an angle θ. θ is the angle by which the boat pitches down at the bow and is also the angle of the axis about which the boat heels. Finally the angle α_R is the angle the rudder is deflected from the boat's centerline.

A sailing boat responding to the forces acting upon it and conforming hydrostatically to deformations in the free surface would heave in the positive Z_o direction an amount s.

The selection of these coordinate systems allows the modeling of forces in either coordinate system. When modeling velocity related forces, such as wave drag and lift, the coordinate system fixed in the water is more attractive computationally. On the other hand, hydrostatic forces are more easily computed in the coordinate system fixed in the

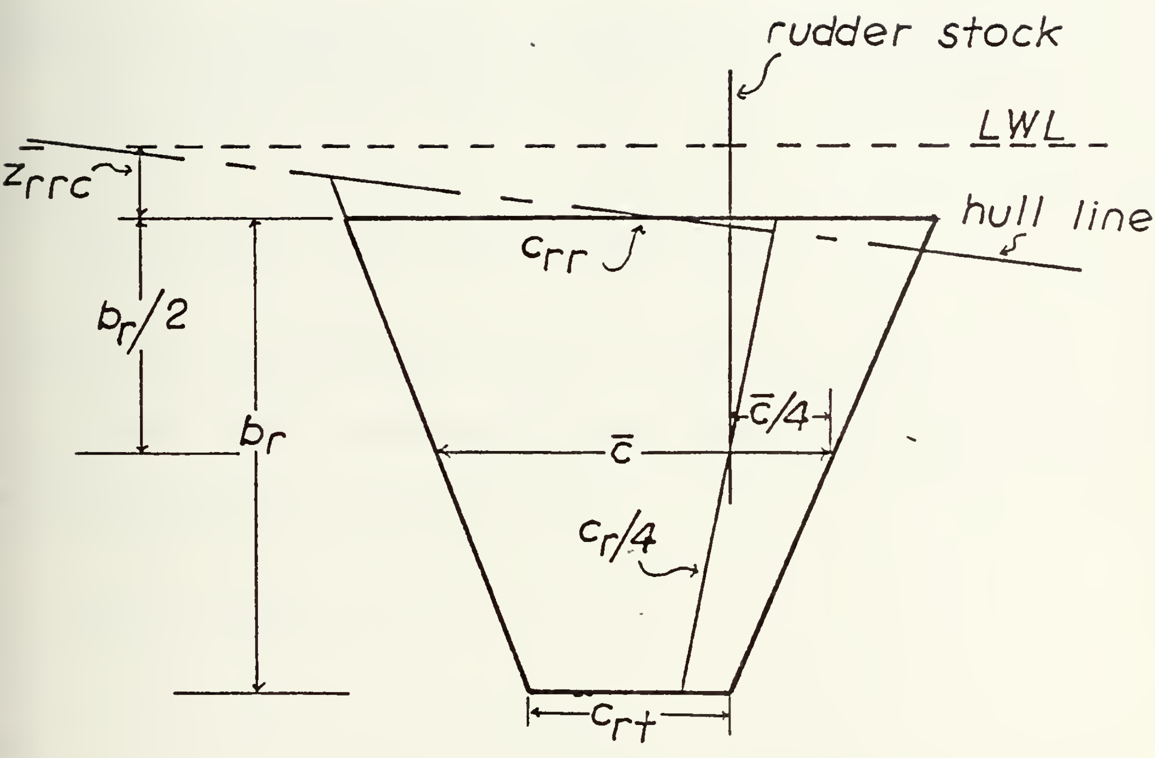

Figure 12 - Rudder Geometry

$\partial C_L/\partial \alpha_r$ is defined (26) as:

$$\frac{\partial C_L}{\partial \alpha_r} = \frac{(0.9)(2\pi)\ a_r}{(57.3)\left[\cos\Omega\sqrt{\frac{a_r^2}{\cos^4\Omega} + 4} + 1.8\right]} \tag{44}$$

where Ω is the sweep angle as defined in Figure 12.

The rudder's drag coefficient, C_{dr}, is defined as:

$$C_{dr} = C_{do} + C_{lr}^2/\pi a_r e + 0.0166\ C_{lr}^2 \tag{45}$$

where e is an efficiency factor established at 0.9 (26). C_{do} is the rudder section form drag coefficient and if not specified C_{do} is assumed to be

$$C_{do} = 0.0065 \tag{46}$$

The 0.0166 C_{lr}^2 term is an emperical increase in functional resistance due to lift recommended by Kerwin (28). This form drag coefficient is from the NACA 0015 section (26). The interference drag caused by the interaction between the fair-body boundary layer and the rudder root chord is not accounted for in the present model and should be examined in future studies.

By transforming the rudder's lift and drag coefficients from the boat's coordinate system into the system located on the horizontal surface of the water, the rudder side force

coefficient is written as:

$$C_{yr} = (C_{lr}\cos\psi - C_{dr}\sin\psi)\cos\phi \qquad 47$$

Hence, the rudder side force is

$$Y_r = -C_{yr}(\tfrac{1}{2}\rho_{H_2O} A_r U_b^2) \qquad 48$$

where A_r is the rudder's plan form area. The plan form area is defined as:

$$A_r = b_r\bar{c}_r \qquad 49$$

Rudder Drag

The rudder drag coefficient is

$$C_{xr} = C_{lr}\sin\psi + C_{dr}\cos\psi \qquad 50$$

where C_{lr} and C_{dr} are defined by equations 42 and 45 respectively.

The equation for rudder drag, X_r, is

$$X_r = -C_{xr}(\tfrac{1}{2}\rho_{H_2O} A_r U_b^2) \qquad 51$$

Rudder Yawing Moment

To determine the yawing moment caused by rudder forces, the location of the center of effort on the rudder, with respect to the rudder stock, is calculated. The rudder stock is assumed to pass through the quarter point on the mean chord as shown in Figure 12.

The moment coefficient about the quarter chord (26) is:

$$C_{m\bar{c}/4} = [0.25 - (\frac{\partial C_m}{\partial C_L})_{C_L=0}] (\frac{\partial C_L}{\partial \alpha_r})_{\alpha_r=0}$$

$$- \frac{1}{2} \frac{C_{dc}}{a_r} (\frac{\alpha_r}{57.3})^2 \qquad 52$$

where

$$(\frac{\partial C_m}{\partial C_L})_{C_l=0} = \frac{1}{2} - \frac{1.11\sqrt{a_r^2+4} +2}{4(a_r+2)} \qquad 53$$

The normal force coefficient, C_n, is

$$C_n = C_{lr}\cos\alpha_r + C_{dr}\cos\alpha_r \qquad 54$$

The location of the center of effort with respect to the quarter point on the mean chord is

$$x_{p\bar{c}/4} = - \frac{C_{m\bar{c}/4}\ \bar{c}}{C_n} \qquad 56$$

The distance from the forward perpendicular to the center of effort of the rudder, x_{cpr}, is

$$x_{cpr} = x_{p\bar{c}/4} + x_{rs} \qquad 57$$

where x_{rs} is the distance from the forward perpendicular to the rudder stock.

The yawing moment due to rudder forces is

$$N_r = x_{cpr} Y_r \qquad 58$$

Rudder Heeling Moment

From reference 26, the distance to the spanwise center of effort from the root chord, z_{ro}, is

$$z_{ro} = \frac{(C_{lr}(4/3\pi) \cos\alpha_r + C_{dr} \sin\alpha_r)b_r}{C_n} . \qquad 59$$

The distance from the waterline to the center of effort is

$$z_{cpr} = z_{ro} + z_{rrc} \qquad 60$$

where z_{rrc} is the distance from the waterline to the root chord as shown in Figure 12.

The rudder heeling moment, K_r, is

$$K_r = z_{cpr} C_{lr} (\tfrac{1}{2}\rho_{H_2O} A_r U_b^2) \qquad 61$$

2.1.4 Hydrostatic Forces

A satisfactory model for the hydrostatic righting moment is presented by Kerwin in reference (5). In modified form, Kerwin's (5) model for the hydrostatic righting moment, N_r, is adopted.

$$N_r = -0.00224[\Delta/(L/100)^3]\ L^4\phi(d_1+d_2\phi+d_3F_n)$$

$$+\ z_g \sin\phi \qquad 62$$

where L is the waterline length, LWL, and d_1, d_2, and d_3 are non-dimensional stability coefficients which depend on hull form. The term, $z_g \sin\phi$ accounts for the distance from the waterline to the center of gravity.

Each coefficient has physical significance and may be input by the model user or generated from the expressions given below. A systematically varied yacht series based on the yacht, Standfast 43, is presently being tested jointly by M.I.T. and Delft University. The hydrostatic data from this series was utilized to generate parameterized expressions for each of the coefficients.

The coefficient, d_1, is the height of the metacenter above the waterline divided by the load waterline length, LWL. An expression for d_1 which accurately predicts the Standfast 43 series is:

$$d_1 = \frac{1119(BWL/L)^3}{[\Delta/(L/100)^3]} - 0.335/(L/T_c) \qquad 63$$

The coefficient, d_2, is the non-linear effect on the righting moment. The emperical expression for d_2 is

$$d_2 = d_1[0.0822(BLW/L) - 0.02162] \qquad 64$$

The coefficient, d_3, quantifies the effect the wave train has on the stability at high Froude Numbers. This effect depends on the boat's beam to length ratio, BWL/L, and the fineness of her bow and stern. The expression for d_3 is an emperical fit to Standfast 43 data and is only dependent on the beam to length ratio.

$$d_3 = [7.8 - 56,350(BWL/L-0.28)^{2.75}\ (10^{-5}) \qquad BWL/L > 0.28 \qquad 65$$

and

$$d_3 = 7.8 \times 10^{-5} \qquad BWL/L \leq 0.28 \qquad 66$$

The location of the center of gravity below the waterline, z_g, may be specified. If not specified, z_g is assumed to be

$$z_g = 0.025L. \qquad 67$$

2.2 Aerodynamic Forces and Moments

The aerodynamic forces are divided into two components, the forces acting on the sailplan and its associated rigging, and the forces acting on the hull.

2.2.1 Sail Forces

The aerodynamic forces are defined in terms of lift and drag. Lift acts at right angles to the apparent wind while drag acts parallel to the apparent wind. These forces are transformed into components fixed in the boat and water coordinate system defined in Figures 1 and 2. Expressions are derived for the sail driving forces, X_s; the sail side force, Y_s; the yawing moment from the sail driving force, N_{sx}; the yawing moment from the sail side force, N_{sy}; and the sail heeling moment, K_s.

Wind Triangle

Computation of the magnitude and direction of the apparent wind is necessary for resolution of the aerodynamic forces. The convention used in this model was adopted by Hazen (6) and defines:

V_{au} = Upright apparent wind velocity

$$= \sqrt{(V_t \sin\gamma)^2 + (U_b + V_t \cos\gamma)^2} \qquad 68$$

and

β_u = Upright apparent wind angle

$$= \arctan[(V_t \sin\gamma/(U_b+V_t \cos\gamma)] \qquad 69$$

where V_t is the true wind velocity and γ is the true wind angle. The trigonometric relationships are graphically depicted in Figure 13. This convention differs from the apparent wind as measured by instruments on a sailing boat by the magnitude of the leeway angle.

In the case of the sail plan, heel angle changes the chord wise direction and magnitude of the apparent wind. Since the spanwise component of the wind makes a negligible contribution to lift and drag, the apparent wind angle and velocity are decreased by the magnitude of the spanwise velocity. The apparent wind velocity is

$$V_a = \sqrt{V_{au}^2[(\cos\phi \sin\beta_u)^2 + \cos^2\beta_u]} \qquad 70$$

and the apparent wind angle is

$$\beta = \arctan\ [(\cos\phi \sin\beta_u)/\cos\beta_u] \qquad 71$$

The resulting lift is normal to the mast.

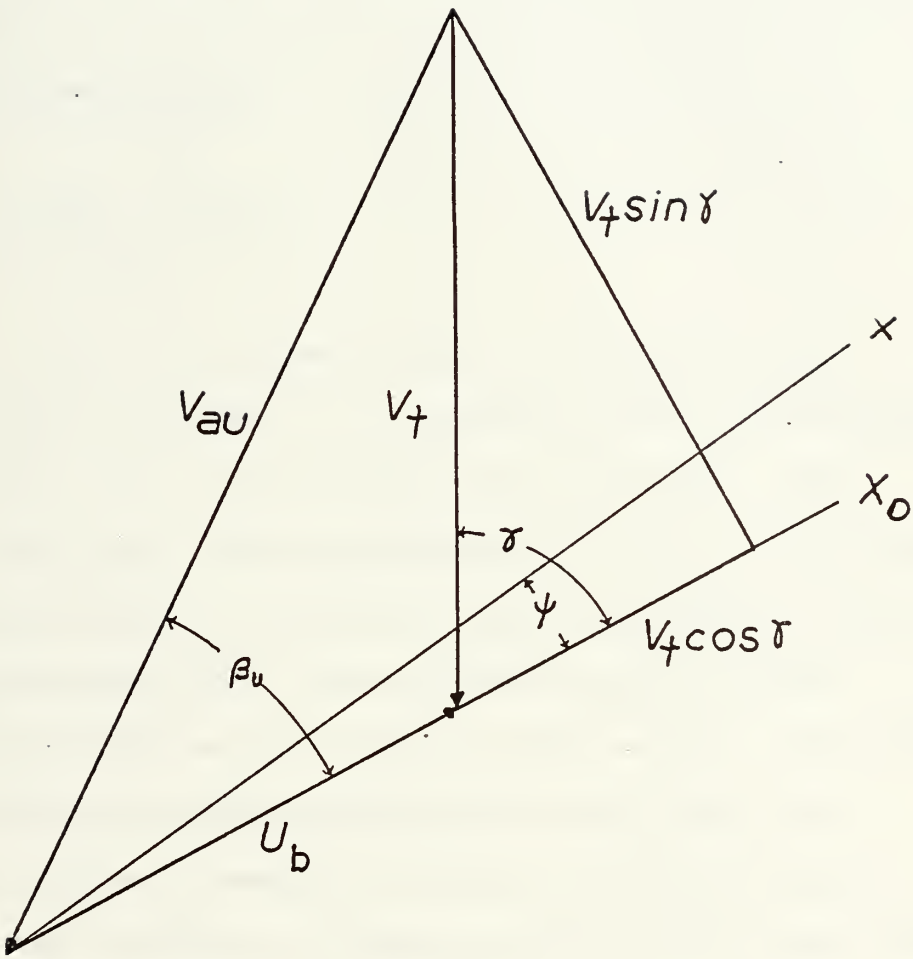

Figure 13 - Apparent Wind Triangle

Lift and Drag

A drawing of a representative sail plan is presented in Figure 14 illustrating the measurements utilized by the sail force model.

Lift is defined as:

$$L_s = C_{ls}(\frac{1}{2}\rho_{air}\ V_a^2\ A_s) \qquad 72$$

where A_s is the total geometric area of the sail plan.

The lift coefficient, C_{ls}, depends on such parameters as sail aspect ratio, boom height, mast diameter, and apparent wind angle. The coefficient of lift is also changed by crew optimization of trim and camber depending on the wind direction and magnitude. The parameterization of the coefficient of lift was not done for this model. The model user may select the lift coefficient or the model will default to 1.25 as a reasonable value. A possible source for lift coefficients as a function of sail plan geometries is given by Milgram (29).

The sail drag may be defined as the summation of form drag, frictional drag, and induced drag. In coefficient form, the expression for sail drag is

$$D_s = (C_{fms}A_m + C_{fs}A_s)\frac{1}{2}\rho_{air}\ V_a^2 + (\frac{C_{dis}}{C_{ls}^2})\ L_s^2/\frac{1}{2}\rho_{air}\ V_a^2 A_s \qquad 73$$

The friction drag coefficient, C_{fs}, has the assumed (29) value of 0.04. The form drag coefficient, C_{fms}, based on the cross-sectional area of the mast, A_m, is a function of the apparent wind angle, β, but to date little data is available on which to base a model. For this reason the form drag coefficient may be specified by the user or default to a constant equal to 0.4. The cross-sectional area of the mast is assumed (29) to be

$$A_m = 0.01(z_p + x_e) \tag{74}$$

where z_p and x_e are defined in Figure 14.

The induced drag of the sail plan is a function of the sail plan geometry. Milgram (29) recommends the use of the ratio C_{dis}/C_{ls}^2 as a measure of sail efficiency. Reference (29) describes a study of systematically varied sail plans using lifting surface theory. From this study, an approximate spline cubic fit of C_{dis}/C_{ls}^2 was established as a function of the aspect ratio of the sail plan, a_s, with a correction for the difference between the aspect ratio of the main, a_m, and the aspect ratio of the fortriangle, a_f. The ratio C_{dis}/C_{ls}^2 is defined as:

$$C_{dis}/C_{ls}^2 = f(a_s) + (a_m - a_f)(0.0625 - 0.00893\ a_s) \tag{75}$$

$$\text{for } a_m > a_f$$

or

$$C_{dis}/C_{ls}^2 = f(a_s) \qquad \text{for } a_m \leq a_f \qquad 76$$

The aspect ratio of the sail, a_s, is defined as:

$$a_s = z_i^2/A_s \qquad 78$$

where z_i is the mast height as shown in Figure 14. The aspect ratio of the main is defined as:

$$a_m = z_p/x_e \qquad 79$$

where z_p and x_e are shown in Figure 14. The aspect ratio of the fortriangle, a_f, is defined as:

$$a_f = z_i/x_j \qquad 80$$

where z_i and x_j are shown in Figure 14. The spline cubic function, $f(a_s)$, is given in Figure 15. The ratio, C_{dis}/C_{ls}^2, is generated for mast head rigs, so the model is capable of accepting the ratio designated by the user. The model may be used for other geometries by selection of coefficients from reference (29).

The above lift and drag coefficients are useful until the apparent wind hauls aft and the jib and shrouds interfere with the main. At this time the sail plan stalls and a different

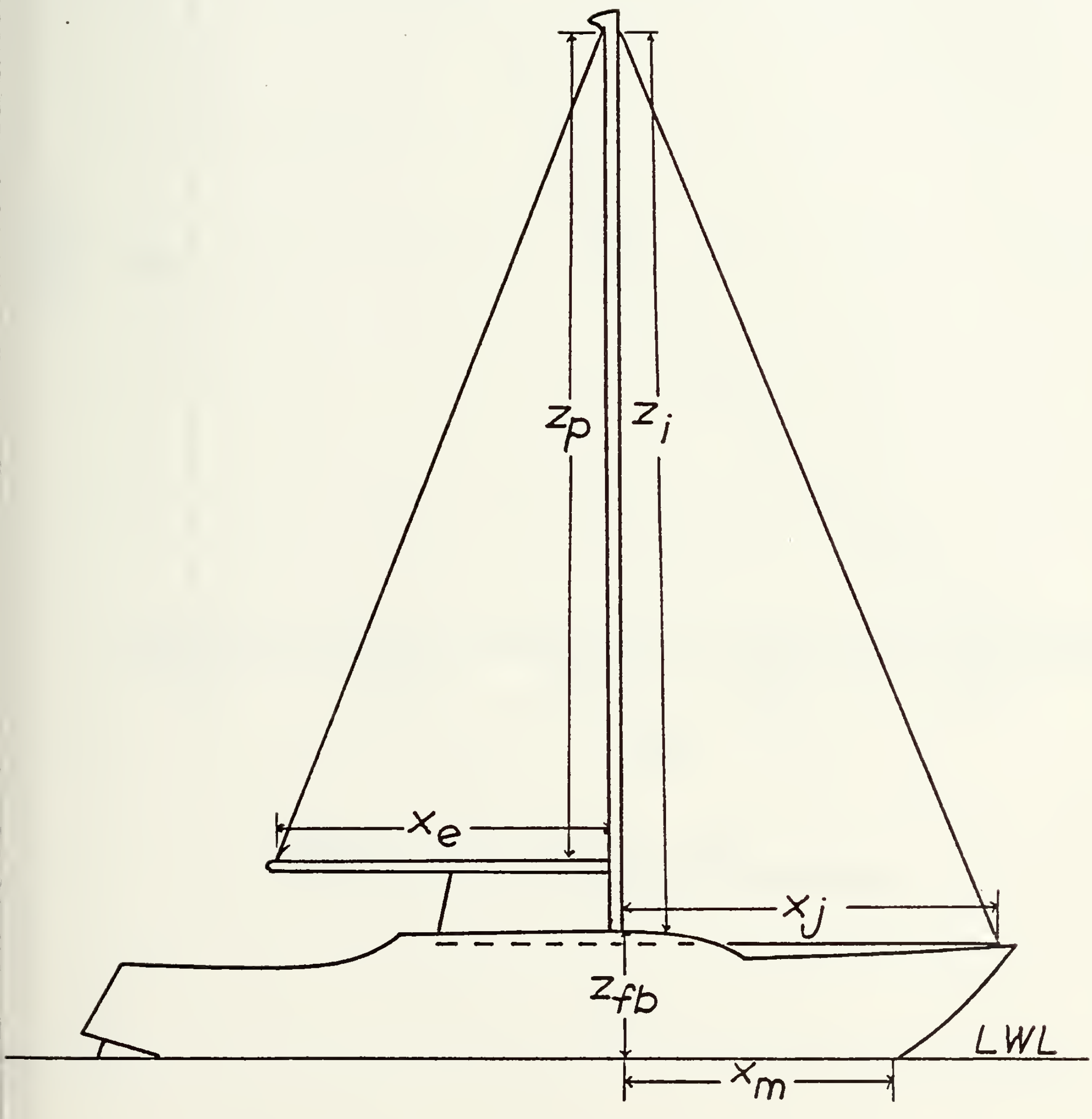

Figure 14 – Sail Plan Measurements

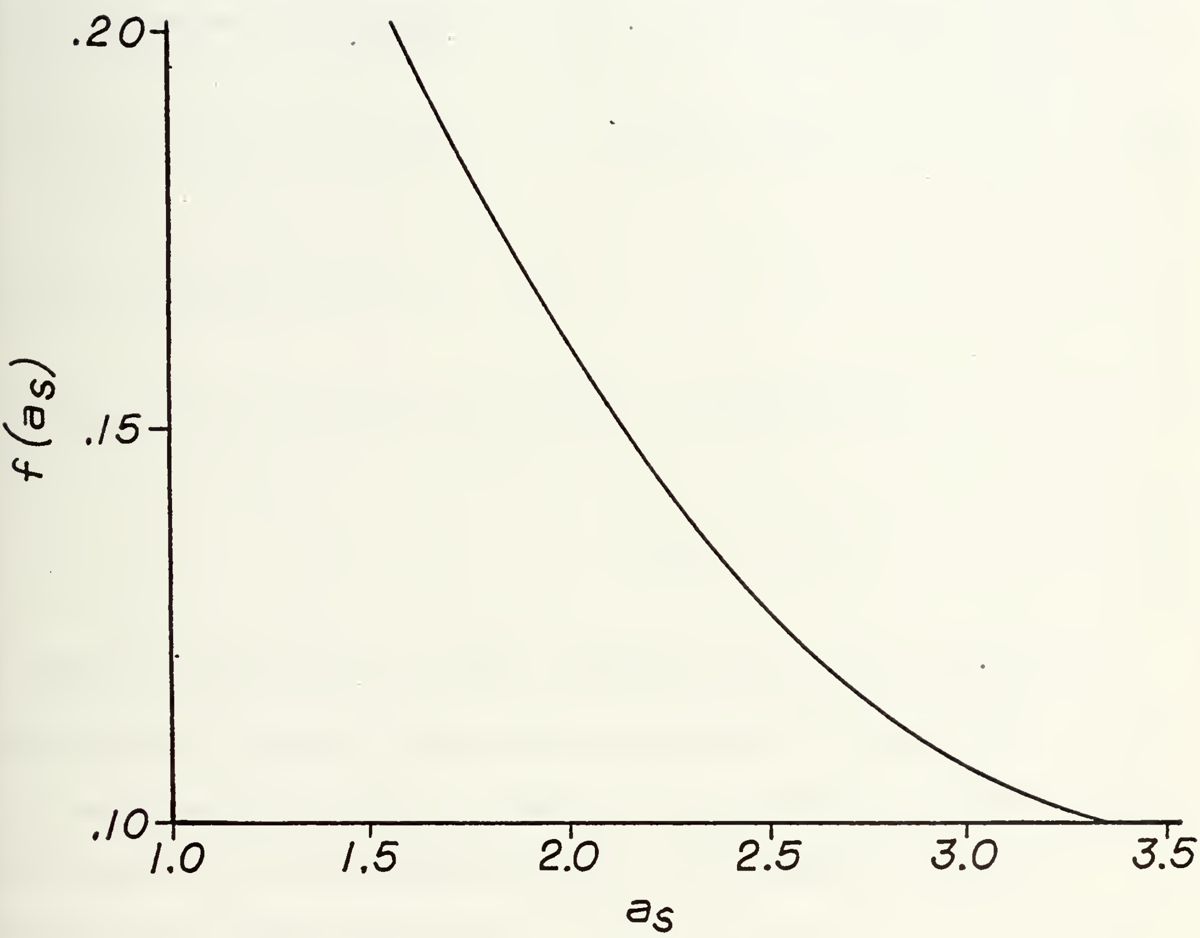

Figure 15 - Sail Efficiency

model must be used between the apparent wind angle of 90° and 180°. Data for systematic variations to sail plans in the off-wind condition is not available. The following simple form for lift and drag when the apparent wind angle, β, is 90° or greater is adopted.

$$L_{so} = L_s \cos(\beta - 90°) \qquad \text{for } \beta \geq 90° \tag{81}$$

and the off-wind drag is

$$D_{so} = D_s + [(1.2/2)\rho_{air} A_s V_a^2 - D_s)\sin(\beta - 90°)$$

$$\text{for } \beta \geq 90° \tag{82}$$

If the off-wind lift and drag coefficients are known for a given point of sail, they may be specified by the user. When data becomes available on reaching the off-wind coefficients as a function of the apparent wind, the function may be incorporated into the model.

2.2.2 Hull Forces

The wind forces on the hull during normal sailing conditions may account for only one or two percent in direct resistance and ten percent of the side force, but as the wind increases and sail is shortened, the hull forces will eventually dominate the system. Very few experiments have been conducted

investigating the effect of wind on ship forms. Hughs (30) conducted tests in water of the topsides of typical tanker, liner, and cargo ship profiles. To date no tests have been conducted on the sailboat form. Hughs (30) did test the above water shape of bare cargo hulls and his emperical results are used to approximate these forces on a sailing boat. The magnitude of the aerodynamic hull force, F_h, is

$$F_h = 0.001428\, V_a^2\, (A_l \sin^2\beta_u + 0.3 A_t \cos^2\beta_u)/\cos(\lambda-\beta) \qquad 83$$

where A_l and A_t are the longitudinal and transverse projected area of the hull respectively. The angle, λ, is the angle between the centerline of the boat and direction from which the hull force acts. The angle λ is taken from the experimental results on hulls reported in reference (30). A spline cubic was fit through this data and is presented in Figure 16.

One effect which cannot be ignored when examining aerodynamic forces on the hull is the vertical wind velocity profile. The hull of a sailing boat sees considerably less wind velocity than the sail plan. Curry (31) measured the wind velocity at different heights and plotted a profile. By assuming a height of two feet above the water surface as representative of an adequate mean for sailing boats, the true wind velocity on the hull was estimated from Curry's wind profile to be 0.55 times the true wind, V_t. The wind

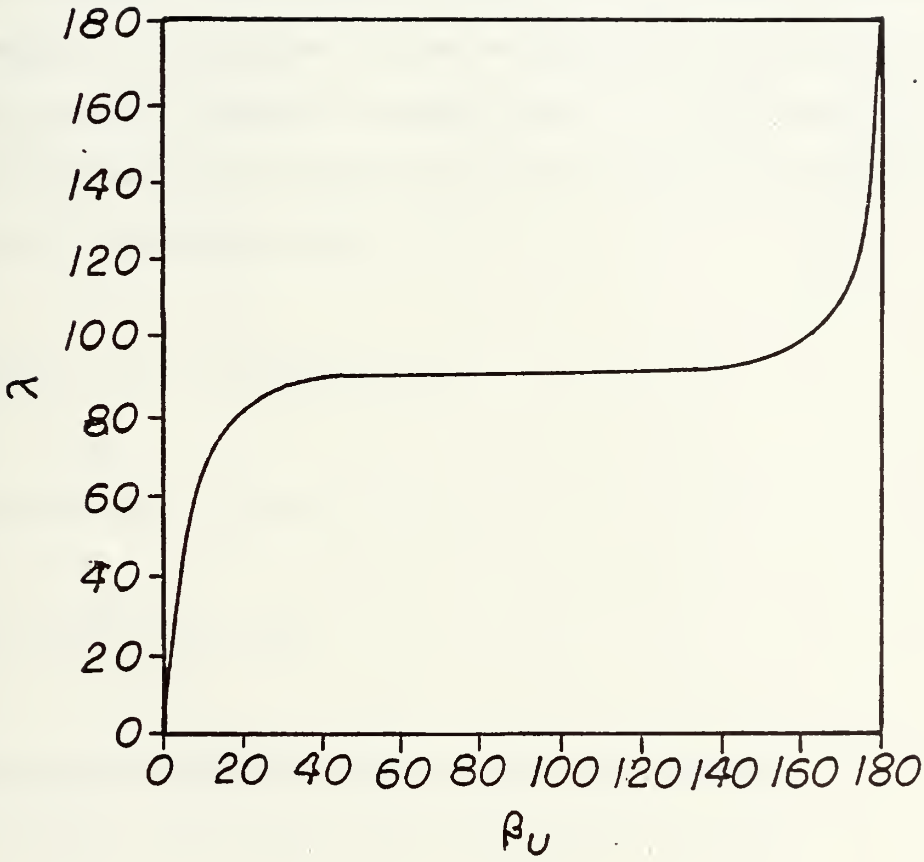

Figure 16 – Hull Force Angle

triangle is calculated with this lower true wind velocity and the hull force is calculated with the resulting apparent wind velocity and angle.

Calculation of the yawing moment requires the location of the center of pressure of the aerodynamic hull force. The distance from the forward perpendicular to the center of pressure divided by the load waterline length, was formulated from data in reference (30).

$$x_{cph}/L = 0.2 + 0.00311\ \beta_u \qquad 84$$

The yawing moment, N_h, is

$$N_h = F_h L (x_{cph}/L) \sin\lambda \qquad 85$$

Resolution of Aerodynamic Forces and Moments

The total aerodynamic driving force, X_s, is

$$X_s = L_s \sin\beta - D_s \cos\beta - F_h \cos\lambda \qquad 86$$

and the side force is

$$Y_s = (L_s \cos\beta + D_s \sin\beta)\cos\phi + F_h \sin\lambda \qquad 87$$

The heeling moment is

$$K_s = Z_{cpys} \left(L_s \cos\beta + D_s \sin\beta\right) + 2.0\, F_h \sin\lambda \qquad 88$$

where z_{cpys} is the distance from the vertical center of effort of the sail side force to the waterline. An expression estimating the location of the vertical center of effort, z_{cpys}, from data in reference (29) is

$$z_{cpys} = 0.43\, z_i + z_{fb} \qquad 89$$

where z_i is the mast height and z_{fb} is the freeboard measured from the waterline to the base of the mast as shown in Figure 14.

The magnitude of the leeway angle is ignored in resolving yawing moments. The yawing moment due to aerodynamic side forces, N_{sy}, is

$$N_{sy} = x_{cps} \left(L_s \cos\beta + D_s \sin\beta\right) \cos\phi + N_h \qquad 90$$

where x_{cps} is the distance from the horizontal center of effort of the sail plan to the forward perpendicular and is defined (29) as:

$$x_{cps} = x_m + 0.02(x_j + x_e) \qquad 91$$

where x_m is the distance from the forward perpendicular to the mast, x_j is the distance from the jib tack to the mast, and x_e is the length of the main boom as shown in Figure 14.

The yawing moment due to the driving force is

$$N_{sx} = z_{cpxs} X_s \sin\phi \qquad 92$$

where z_{cpxs} is the distance from the vertical center of the driving force to the center of resistance. z_{cpxs} is estimated by

$$z_{cpxs} = 0.48 z_i + z_{fb} \qquad 93$$

III. GEOMETRY SYNTHESIS

3.1 Parameter Generation

The computerized version of the mathematical model has the capability to generate dimensions of a satisfactory design from which parametric trade-offs may be made. The user of the program may specify as many or as few of the dimensions as he desires. This section presents the equations which determine the unspecified dimensions.

The equations do not generate an optimum design. The majority of the expressions are estimates by the author of the dimensions of a typical ocean racer/cruiser and no other justification is made for their formulation.

If the waterline length is unspecified, the length is set at:

$$LWL = 30.0 \text{ feet} \qquad 94$$

The default displacement to length ratio, $\Delta/(L/100)^3$, is

$$\Delta/(L/100)^3 = 225 \qquad 95$$

and the prismatic coefficient, C_p, is

$$C_p = 0.53 \qquad 96$$

The beam to length ratio, BWL/LWL, is defined by a linear fit through the mean of a large sample of boats presently measured under the International Offshore Rule. The beam to length ratio is defined by:

$$BWL/LWL = 0.3633 - 0.002018\ LWL \qquad 97$$

By assuming that the base boat has a constant block coefficient, C_b, the length to fairbody draft ratio may be written as:

$$LWL/T_c = 10975\ (BWL/LWL)/[\Delta/(L/100)^3] \qquad 98$$

As a percentage of the waterline length, the longitudinal position of the center of buoyancy, LCB, is

$$LCB = 53.0 \qquad 99$$

The fluid properties are established at the following values:

kinematic viscosity of water:

$$\gamma_{H_2O} = 1.2791 \times 10^{-5} \qquad 100$$

density of water:

$$\rho_{H_2O} = 1.99 \qquad 101$$

density of air:

$$\rho_{air} = 0.00238 \qquad 102$$

The mean chord of the keel, $\bar{c}_k$, is estimated by:

$$\bar{c}_k = 0.23\ LWL \qquad 103$$

The form resistance and correlation coefficient, C_{fm}, is assumed to be:

$$C_{fm} = 0.0 \qquad 104$$

The estimate of fairbody wetted surface gives a satisfactory prediction of the Standfast 43 Series and predicts Antiope's fairbody wetted surface to within 1.5%. For most modern yacht forms, the expression should give a satisfactory estimate for preliminary design purposes. The expression for fairbody wetted surface, S_{fb}, is

$$S_{fb} = (LWL)^2\{1.7/(LWL/T_c)+3.5\text{x}10^{-5}[\Delta/(L/100)^3]\ (LWL/T_c)\}$$

$$\{0.0712[(BWL/T_c)-4.0] + 1.0\} \qquad 105$$

The sail area, A_s, is established at

$$A_s = (LWL)^2 \qquad 106$$

The dimensions of the sailplan, as described in Figure 14, are defined by assuming the aspect ratio of the fortri-angle, a_f, and the aspect ratio of the main, a_m, are equal to three. The foot of the jib is assumed to be 160% of the base of the fortriangle. It follows that the mast height is

$$z_i = \sqrt{A_s/0.4044} \tag{107}$$

and the base of the fortriangle is

$$z_j = z_i/3.0 \tag{108}$$

The main hoist length, z_p, is assumed to be

$$z_p = z_i/1.1 \tag{109}$$

and the main boom then is

$$z_e = z_p/3.0 \tag{110}$$

The horizontal positions of the profile measurements as defined in Figure 9 are as follows:

$$x_o = 0.0 \tag{111}$$

$$x_1 = 0.1\ \mathrm{LWL} \tag{112}$$

$$x_2 = 0.2 \text{ LWL} \tag{113}$$

$$x_3 = 0.3 \text{ LWL} \tag{114}$$

$$x_b = 0.55 \text{ LWL} \tag{115}$$

The maximum draft, b_o, is assumed to be the base draft as defined by the International Offshore Rule (33) and is calculated by:

$$b_o = 0.146 \text{ LWL} + 2.0 \tag{116}$$

At the other previously defined horizontal positions, the draft measurements are

$$b_1 = 0.98\ b_o \tag{117}$$

$$b_2 = 0.95 \text{ LWL}/(\text{LWL}/T_c) \tag{118}$$

$$b_3 = 0.7 \text{ LWL}/(\text{LWL}/T_c) \tag{119}$$

$$b_b = 0.0 \tag{120}$$

The ratio, r_o/b_o, as defined in Figure 9, is

$$r_o/b_o = b_o \text{ LWL}/(\text{LWL}/T_c) \tag{121}$$

The sailplan is positioned in the boat so that the center of effort of the sailplan, x_{cps}, coincides with the center of effort of the keel, x_{cpk}, as defined in equation 37. The horizontal distance from the forward perpendicular to the sail's center of effort, x_{cps}, is defined as:

$$x_{cps} = x_{cpk} \qquad 122$$

The forward overhang, FOH, is not determined by asthetics, but coincides with the placement of the jib tack. This point is calculated by the expression:

$$FOH = 0.02\ (x_j + x_e) + x_j - x_{cps} \qquad 123$$

The freeboard, z_{fb}, is derived from the base freeboard as defined by the International Offshore Rule (33) with a constant added to account for the height of the cabin top as shown in Figure 14. The freeboard is defined as:

$$z_{fb} = 0.057\ LWL + 2.0 \qquad 124$$

An estimation of the projected lateral area of the hull, A_l, is

$$A_l = (2.1\ LWL + FOH) z_{fb}/2.0 \qquad 125$$

and the project transverse area of the hull, A_t, is estimated by the equation:

$$A_t = 1.1\ LWL(BWL/LWL)z_{fb} \qquad 126$$

The wetted surface of the keel, S_k, is estimated by:

$$S_k = 2.1\bar{c}_k\ [b_o - LWL/(LWL/T_c)] \qquad 127$$

The dimensions of the rudder are determined by the following equations. The distance from the forward perpendicular to the rudder stock, x_{rs}, is

$$x_{rs} = 0.95\ LWL \qquad 128$$

and the distance from the waterline to the root chord is

$$z_r = 0.2\ LWL/(LWL/T_c) \qquad 129$$

The root chord of the rudder is defined by:

$$c_{rr} = 0.075\ LWL \qquad 130$$

and by assuming a taper ratio, the tip chord is given by:

$$c_{rt} = 0.667\ c_{rr} \qquad 131$$

The mean chord follows as:

$$\bar{c}_r = (c_{rr} + c_{rt})/2.0 \qquad 132$$

An estimate of the span of the rudder is given by the expression,

$$b_r = 0.85(b_o - z_r) \qquad 133$$

and the sweep angle, Ω, is set to zero,

$$\Omega = 0.0 \qquad 134$$

The rudder area, A_r, is

$$A_r = b_r\bar{c}_r \qquad 135$$

and the rudder's wetted surface is estimated by:

$$S_r = 2.1\ A_r \qquad 136$$

IV. PROCEDURE

4.1 Equilibrium Computation

To find the equilibrium solution of the forces and moments for a given wind direction and speed, equations 1 through 4 must be solved. The equations are rewritten as:

$$R_1 = X_s + X_h + X_k + X_r = 0 \qquad 137$$

$$R_2 = Y_s + Y_k + Y_r = 0 \qquad 138$$

$$R_3 = K_s + K_h + K_k + K_r = 0 \qquad 139$$

$$R_4 = N_{sx} + N_{sy} + N_k + N_r = 0 \qquad 140$$

The solution of these equations require a technique for solving non-linear equations. The technique utilized is Newton-Raphson Iteration described in references (34) and (6).

$\overline{R}$ is an "error" vector defined by the components R_1 through R_4,

$$\overline{R} = [R_1, R_2, R_3, R_4] \qquad 141$$

where the vector, $\overline{R}$, is equal to zero when the sailing boat is in force and moment equilibrium. Due to the iterative nature of the Newton-Raphson technique, the value of the vector, $\overline{R}$, is minimized to an acceptable tolerance.

$\overline{P}$ is defined as the vector of assumed values of the four independent variables,

$$\overline{P} = [U_b,\ \psi,\ \phi,\ \alpha_r] \qquad 142$$

and the first partial derivative of the component "errors" are defined as:

$$
\begin{aligned}
s_{11} &= \frac{\partial R_1}{\partial P_1}, & s_{12} &= \frac{\partial R_1}{\partial P_2}, & s_{13} &= \frac{\partial R_1}{\partial P_3}, & s_{14} &= \frac{\partial R_1}{\partial P_4} \\
s_{21} &= \frac{\partial R_2}{\partial P_1}, & s_{22} &= \frac{\partial R_2}{\partial P_2}, & s_{23} &= \frac{\partial R_2}{\partial P_3}, & s_{24} &= \frac{\partial R_2}{\partial P_4} \\
s_{31} &= \frac{\partial R_3}{\partial P_1}, & s_{32} &= \frac{\partial R_3}{\partial P_2}, & s_{33} &= \frac{\partial R_3}{\partial P_3}, & s_{34} &= \frac{\partial R_3}{\partial P_4} \\
s_{41} &= \frac{\partial R_4}{\partial P_1}, & s_{42} &= \frac{\partial R_4}{\partial P_2}, & s_{43} &= \frac{\partial R_4}{\partial P_3}, & s_{44} &= \frac{\partial R_4}{\partial P_4}
\end{aligned}
\qquad 143
$$

These partial derivatives are computed by taking first differences.

By defining $\overline{\delta}$ as a vector of required incremental changes to the independent variables, a matrix of linearized equations for each "error", R_1 through R_4 is established.

$$
\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix}
\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix}
=
\begin{bmatrix} -R_1 \\ -R_2 \\ -R_3 \\ -R_4 \end{bmatrix}
\qquad 144
$$

Therefore:

$$[\delta_1, \delta_2, \delta_3, \delta_4]^T = \begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{pmatrix}^{-1} \begin{pmatrix} -R_1 \\ -R_2 \\ -R_3 \\ -R_4 \end{pmatrix} \qquad 145$$

The new values of the independent variables becomes:

$$P_1(i+1) = P_1(i) + \delta_1 \qquad 146$$

$$P_2(i+1) = P_2(i) + \delta_2 \qquad 147$$

$$P_3(i+1) = P_3(i) + \delta_3 \qquad 148$$

$$P_4(i+1) = P_4(i) + \delta_4 \qquad 149$$

Convergence is tested by examining the magnitude of the component "errors", and the vector, $\overline{P}$, is updated until the desired tolerance is satisfied.

V. RESULTS

Figures 17 and 18 are performance polar plots of two yachts. Figure 17 shows the theoretical boat velocity of the yacht Antiope as predicted by the mathematical model. The velocity is plotted as a function of the true wind angle, γ, and the three wind velocities 5, 7.5, and 10 knots. For this prediction the sail area was assumed to be 400 square feet and the center of gravity is 0.61 feet below the waterline. The sailplan geometry and placement was generated by the model and the keel fixed rudder was modeled by placing the program's spade rudder at the trailing edge of the keel and setting the wetted surface of the rudder equal to zero. Since the correlation of many of the model's forces are derived from data of the full scale testing of Antiope (3), her performance polar is included in the results even though the model is not designed for accurate prediction of rudder forces on keel mounted rudders.

Figure 18 gives the performance polar of the model generated sailing boat for the same three wind velocities. The geometry of each boat is fully described in the computer printout listed in Section 8.3. The two example output listings in Section 8.3 are the source for the plots in Figures 17 and 18.

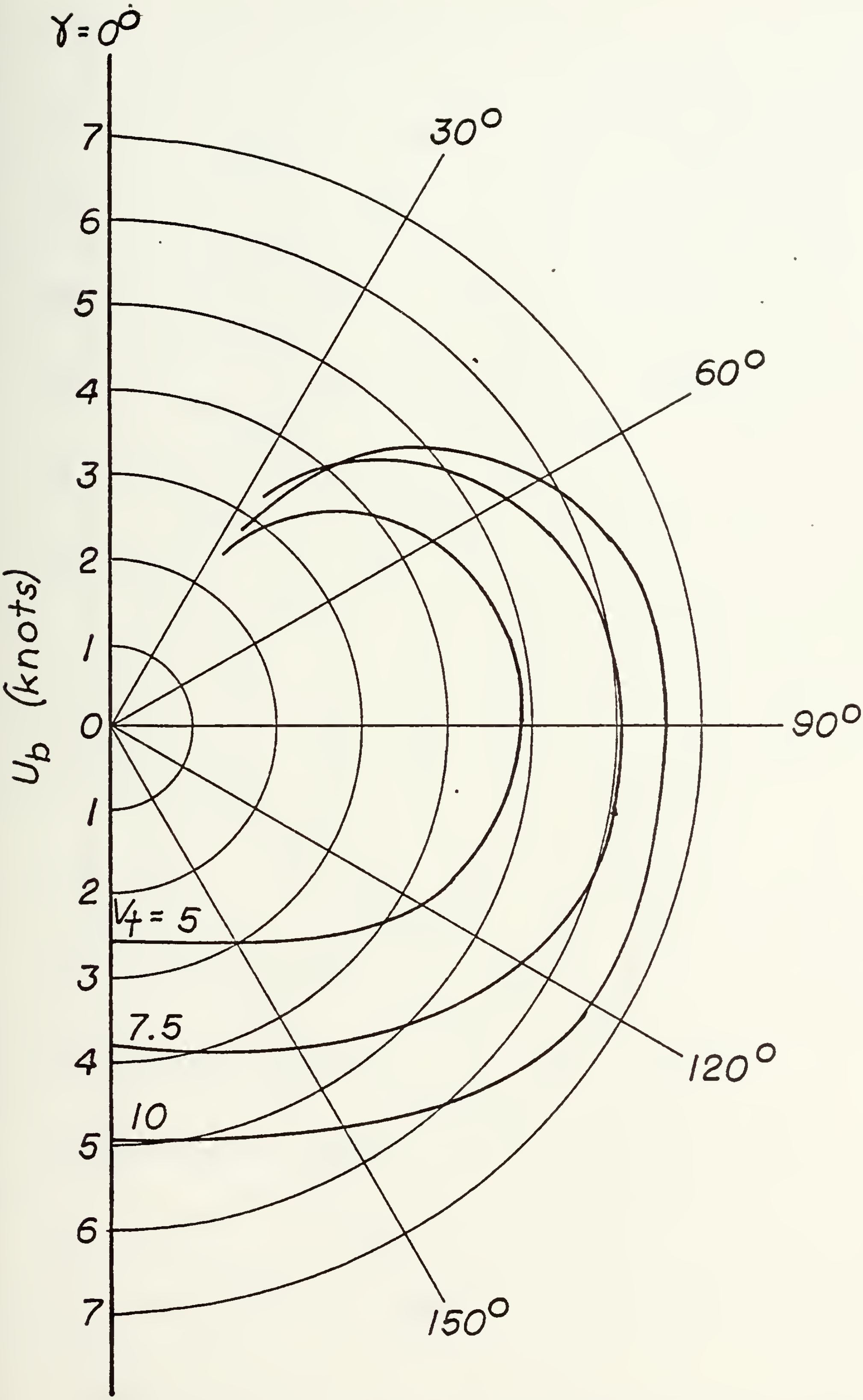

Figure 17 - Antiope Performance Polar

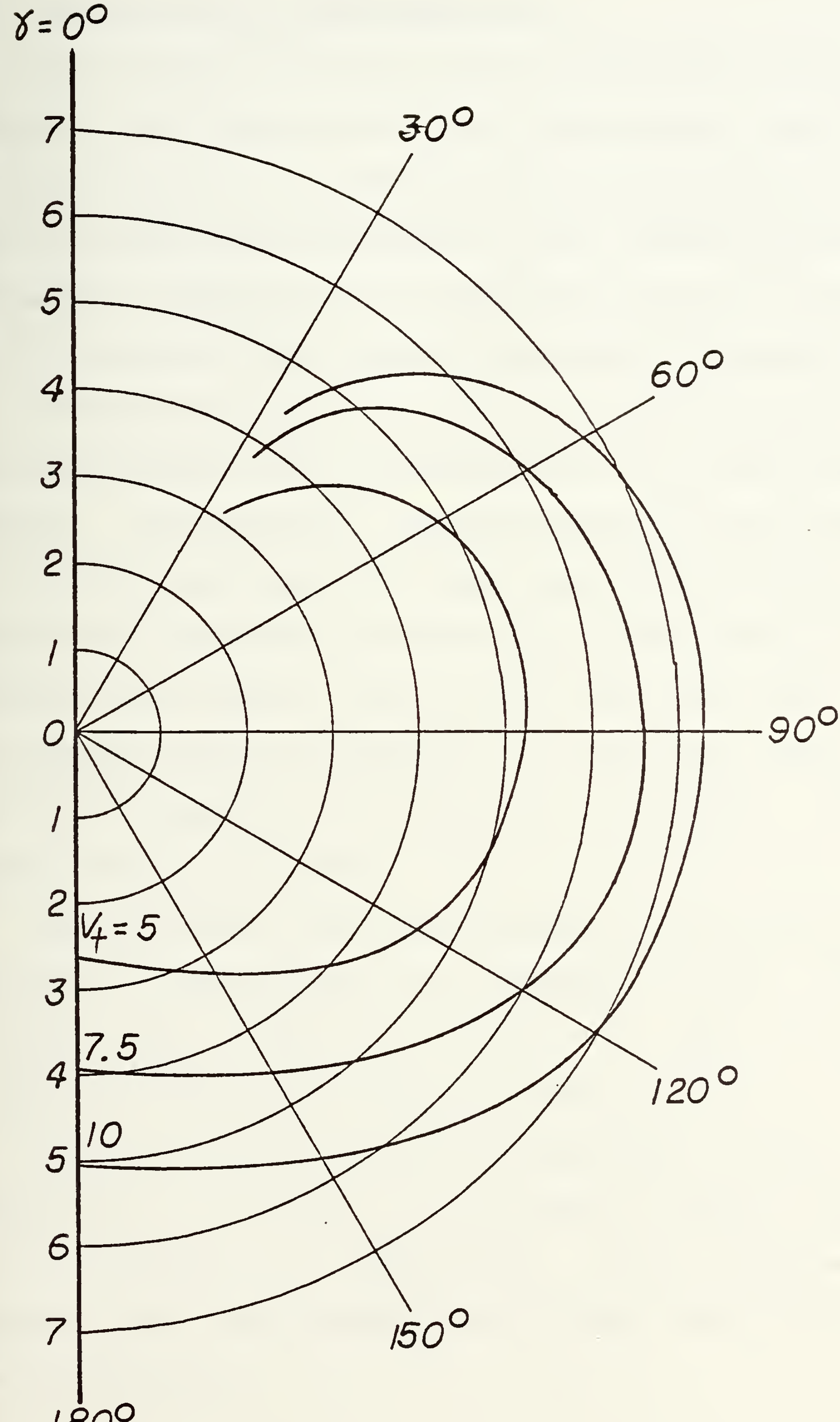

Figure 18 - Default Boat Peformance Polar

VI. CONCLUSIONS AND RECOMMENDATIONS

Two important criteria should be considered when evaluating the capabilities of a system model. The first is the verification of the accuracy of the predicted performance, and the second is an assessment of the capabilities and limitations a model has for predicting changes in performance due to variation in the controlling geometric parameters.

To determine the accuracy of the model in predicting total performance, the performance data of a known yacht is required. Unfortunately, the measurement of the performance of an actual yacht under sail has never been accomplished. To provide a basis for correlation, the four independent variables, boat speed, leeway angle, heel angle, and rudder angle would have to be measured on a full scale yacht along with true or apparent wind velocity and direction. This data is not available to date.

Other programs exist for resolving tow tank data and assumed sail coefficients into performance polars. These programs might be used in the future as a basis for comparison if large or full scale towing tank data is used.

The use of other performance prediction programs for correlation is not useful unless another standard exists with which to measure their capabilities. If two performance programs agree, the agreement may be that both will give the same wrong answer.

The polar plots in Figures 17 and 18 are considered reasonable only because they correlate subjectively with the author's own sailing experience. Though each component force is correlated against either emperical or theoretical data, a means of examining the capability of the mathematical model for the total system should be developed in the future.

The capability of a mathematical model to respond correctly to changes in boat geometry and therefore enable parametric tradeoffs requires physical accuracy of the individual force models. When the individual force models were presented, the limitation and capabilities of each was discussed. The limitations and capabilities of each model are summerized below.

Hull Resistance

Hull resistance is calculated by superposition of frictional resistance, wave resistance and form resistance.

The frictional resistance coefficient is based on the ITTC line and is applied to the fairbody, the keel and the rudder. This line includes an unspecified amount of form resistance. Future model refinement should include a frictional coefficient which does not include form resistance and a mathematical model that parameterizes form resistance. The form resistance coefficient is not parameterized in the present model.

Wave resistance is estimated by a combination of the results of Michell's theory, Taylor Standard Series tests and full scale tests of the yacht Antiope. The resulting semi-emperical mathematical model for wave resistance needs to be compared to the results of a series of full scale yacht wave resistance tests.

The mathematical model for hull resistance is considered adequate for predicting calm water resistance. To predict a boat's performance in rough water, a mathematical model predicting the added resistance due to waves should be developed.

The change in resistance due to heel angle is also not parameterized and was modeled as an average of what good designs can expect. The investigation and incorporation of heel effects would be worthwhile.

The present resistance model is considered dependable for parametric variation in displacement to length ratio and wetted surface.

Keel Forces

The hydrodynamic side force on the hull and keel are derived from slender-body theory modified for fish-like forms (24). This model provides accurate prediction of forms fixed with moderate keels such as found on the yacht Antiope. A method for determining decrease in lift as the keel chord decreases at a given span should be developed. The present

method adequately models changes in performance due to draft but is insensitive to reduction in keel area due to chord length changes.

The interference effects between the hull boundary layer and the root chord of the keel should be the subject of further investigation.

Rudder Forces

The rudder forces are modeled from the free stream tests of low and medium aspect ratio lifting surfaces (26). The model is adequate for yachts with spade rudders, but many yachts are being designed with skeg rudders. A model should be developed for rudder forces as a function of the percentage flap on the skeg rudder. The fairbody boundary layer interference effects on the rudder have been ignored in the present model.

The mathematical model is expected to satisfactorily model changes in the spade rudder geometry.

Hydrostatic Forces

Hydrostatic forces are modeled using the concept of the metacentric height and are adjusted for Froude Number effects. The model is considered sensitive to geometric changes though further verification of the Froude Number effects is desirable.

To accurately model the righting moment on smaller yachts, a function representing the effect of crew weight on righting moment should be added.

Sail Forces

The sail force mathematical model is based on lifting surface theory presented in reference (29). The model is adequate for close hauled prediction but is not parameterized for reaching and running due to lack of data. Work is needed to determine force coefficients as a function of the apparent wind angle and sail plan geometry for off wind conditions. Further work needs to be accomplished to parameterize the form drag coefficient as a function of rig geometry and apparent wind angle.

The utilization of the model should be limited to light and moderate wind conditions until the optimization variables for reefing and flattening are included.

The effect of the vertical wind gradient on heel has not been included and should be added in the future. When added resistance in waves is investigated, it will also be important to study the effect large pitching motions have on sail forces and their relationship to the added resistance in waves.

The limitations of the sail force model in its present form preclude the accurate prediction in off wind and heavy air conditions.

Aerodynamic Hull Forces

The wind forces acting on the hull are modeled from data on cargo ship hulls (30). Though the model is derived from a different hull form, the prediction of these forces

is considered reasonable. Tests on yacht hulls would help improve the correlation for this mathematical model, but the effect on the final prediction should be minor.

In Conclusion

The present form of the mathematical models is useful for preliminary design in determining the quantitative effect changes in the major geometrical parameters have on a boat's performance. These parameters include displacement to length ratio, wetted surface, beam to length ratio, total draft, rudder geometry, position of the center of gravity, sail plan geometry for on wind conditions, and sail area.

The development of the model has been useful in cataloguing available theoretical and emperical sources for modeling forces acting on a sailing boat and has made clear the necessary improvements required in the science of sailing boats to improve prediction capability. For the naval architect, this first generation system model for sailing boat performance may be used as a file to which improvements may be made as a greater understanding of the science develops.

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VIII. APPENDICES

8.1 Computer Input

The required data cards are divided into two data sets described below:

Card Set Number One

Card set number one consists of the input points necessary to generate the spine cubic curves required by the force models. There are twenty-four cards punched in FORMAT (8F10.0). This data set is an integral part of the program and does not change from run to run. Table 1 is a listing of the required data cards.

Card Set Number Two

Card set number two is repeated for each boat evaluated by the mathematical model. The cards control the program options and describe the geometry of a given sailing boat. A listing of the card set used to describe the yacht Antiope is given in Table 2. Table 3 is an example of the required data set where all parameters are defaulted.

Card Set #2

card #1	IFAM, NGAM, NVT; FORMAT (8I10); no default.
col 10	If IFAM = 1 forces and moments are printed.
	If IFAM = 0 no forces and moments are printed.
col 11 - 20	NGAM = number of true wind angles desired.
col 21 - 30	NVT = number of true wind velocities desired.

card #2	ITMAX, IPRINT, N; FORMAT (10X, I3, 17X, I1, 19X, I3); no default.
col 11 - 13	ITMAX = maximum number of iterations allowed per solution.
col 31	If IPRINT = 1, the independent variables, apparent wind data, and forces and moments will be printed out each iteration. If IPRINT = 0, only the convergent or final iteration will be printed.
col 53	N = 4, the number of degrees of freedom
card #3	EPS1, EPS2; FORMAT (10X, E7.1, 13X, E7.1); no default.
col 11 - 17	EPS1 = required tolerance in force equilibrium.
col 31 - 37	EPS2 = step size taken for computing first differences.
card #4	(XOLD(I), I=1,4); FORMAT (20X, 5F10.3); no default.
col 21 - 30	XOLD(1) = initial assumed value of boat velocity, (feet per second)
col 31 - 40	XOLD(2) = initial assumed heel angle, (degrees)
col 41 - 50	XOLD(3) = initial assumed leeway angle, (degrees)
col 51 - 60	XOLD(4) = initial assumed rudder angle, (degrees)
card #5 or more	(KVT(I),I = 1,NVT); FORMAT (8F10.5); no default.
col 1 - 80	KVT(I) = desired true wind velocities (knots)
card #6 or more	(DGAMMA(I), I = 1,NGMA); FORMAT (8F10.5), no default
col 1 - 80	DGAMMA(I) = desired true wind angles (degrees)

card #7	C(2), C(7), C(8), C(9), C(10); FORMAT (8F10.5); default = -1.0
col 1 - 10	C(2) = waterline length (feet
col 11 - 20	C(7) = displacement to length ratio
col 21 - 30	C(8) = prismatic coefficient
col 31 - 40	C(9) = length to draft ratio
col 41 - 50	C(10) = distance from FP to LCB (%LWL)
card #8	C(11), FORMAT (E14.7), default = -1.0
col 1 - 14	C(11) = kinematic viscosity of water
card #9	(C(I), I = 12,18); FORMAT (8F10.5); default = -1.0
col 1 - 10	C(12) = density of water
col 11 - 20	C(13) = mean keel chord (feet)
col 21 - 30	C(14) = mean rudder chord (feet)
col 31 - 40	C(15) = form and correlation coefficient
col 41 - 50	C(16) = fairbody wetted surface
col 51 - 60	C(17) = keel wetted surface
col 61 - 70	C(18) = rudder wetted surface
card #10	C(1), C(3), C(4), C(5), C(6); FORMAT (8F10.5); default = -1.0
col 1 - 10	C(1) = beam to length ratio
col 11 - 20	C(3) = sail area
col 21 - 30	C(4) = heeling moment arm
col 31 - 40	C(5) = pitching moment arm
col 41 - 50	C(6) = distance from FP to rudder stock

card #11	(C(I),I = 19,26); FORMAT (8F10.5); default = -1.0
col 1 - 10	C(19) = close hauled lift coefficient
col 11 - 20	C(20) = rig form drag coefficient
col 21 - 30	C(21) = cross sectional area of rig
col 31 - 40	C(22) = sail frictional coefficient
col 41 - 50	C(23) = induced drag coef/lift coef^2
col 51 - 60	C(24) = lateral area of the hull
col 61 - 70	C(25) = transverse area of the hull
col 71 - 80	C(26) = distance from CE sail to FP
card #12	(C(I),I = 27,30); FORMAT (8F10.5); default = -1.0
col 1 - 10	C(27) = off wind drag coefficient
col 11 - 20	C(28) = off wind lift coefficient
col 21 - 30	C(29) = distance for CE hull to FP
col 31 - 40	C(30) = density of air
card #13	(C(I), I = 31,38); FORMAT (8F10.5); default = -1.0
col 1 - 10	C(31) = x_0
col 11 - 20	C(32) = x_1
col 21 - 30	C(33) = x_2
col 31 - 40	C(34) = x_3
col 41 - 50	C(35) = x_b
col 51 - 60	C(36) = b_0
col 61 - 70	C(37) = b_1
col 71 - 80	C(38) = b_2

card #14	C(39), C(40); FORMAT (8F10.5); default=-1.0
col 1 - 10	C(39) = b_3
col 11 - 20	C(40) = b_b
card #15	(C(I), I = 41,43); FORMAT (8F10.0); default = -1.0
col 1 - 10	C(41) = draft of the canoe body/total draft
col 11 - 20	C(42) = hydrodynamic lift coefficient
col 21 - 30	C(43) = ψX lift/drag
card #16	(C(I), I = 45,52); FORMAT (8F10.5); default = -1.0
col 1 - 10	C(45) = distance from LWL to rudder root chord
col 11 - 20	C(46) = rudder root chord
col 21 - 30	C(47) = rudder tip chord
col 31 - 40	C(48) = rudder span
col 41 - 50	C(49) = rudder sweep angle
col 51 - 60	C(50) = rudder aspect ratio
col 61 - 70	C(51) = rudder mean chord
col 71 - 80	C(52) = rudder $\partial C_l/\partial \alpha_r$
card #17	(C(I), I = 53,56), C(61); FORMAT (8F10.5); default = -1.0
col 1 - 10	C(53) = rudder cross flow drag coefficient
col 11 - 20	C(54) = rudder $\partial C_m/\partial C_l$
col 21 - 30	C(55) = rudder form drag coefficient
col 31 - 40	C(56) = rudder area
col 41 - 50	C(61) = distance from CE keel to LWL

card #18	(C(I), I = 57,60); FORMAT (8F10.5); default = -1.0
col 1 - 10	C(57) = metacentric height/LWL
col 11 - 20	C(58) = Froude Number righting moment coefficient
col 21 - 30	C(59) = non-linear hydrostatic coefficient
col 31 - 40	C(60) = distance from LWL to CG
card #19	(C(I), I = 62,69); FORMAT (8F10.5); default = -1.0
col 1 - 10	C(62) = mast height
col 11 - 20	C(63) = base of the fortriangle
col 21 - 30	C(64) = main hoist
col 31 - 40	C(65) = main boom length
col 41 - 50	C(66) = aspect ratio of the fortriangle
col 51 - 60	C(67) = aspect ratio of the main
col 61 - 71	C(68) = aspect ratio of the sail area
col 71 - 80	C(69) = distance from mast aft to sail CE
card #20	C(70), C(71); FORMAT (8F10.5); default=-1.0
col 1 - 10	C(70) = forward overhang
col 11 - 20	C(71) = freeboard
card #21	KEND, FORMAT (I1), no default
col 1	If KEND = 1, indicates this is final data set
	If KEND = 0, indicates other data sets to follow

0.0	0.2	0.4	0.6	0.7	0.8	0.9	1.0
1.1	1.2	1.25					
0.0	0.00006	0.00011	0.000277	0.0028	0.00703	0.01425	0.0349
0.0549	0.07196	0.0898					
0.0	0.0011	0.0022	0.0033	0.00385	0.006	0.008	0.01
0.008	0.006	0.005					
0.0	0.007	0.012	0.02	0.025	0.03	0.046	0.0821
0.0467	0.0326	0.029					
0.53	0.53	0.53	0.53	0.53	0.54	0.545	0.535
0.547	0.58	0.60					
0.0	0.08767	0.17453	0.34907	0.5236	0.69813	0.87266	1.0472
1.2217	1.3963	1.5708	1.7453	1.9199	2.0944	2.2689	2.4434
2.618	2.7925	2.9671	3.0543	3.1416			
0.0	0.8378	1.1345	1.4312	1.5359	1.5533	1.5708	1.5708
1.5708	1.5708	1.5708	1.5708	1.5708	1.5708	1.5795	1.6057
1.6406	1.6929	1.8326	2.0595	3.1416			
0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
0.8	0.9	1.0					
1.0	0.98	0.95	0.885	0.8	0.68	0.535	0.385
0.22	0.09	0.0					
2.0	2.002	2.012	2.042	2.095	2.184	2.33	2.555
3.0	5.0	10.0					
1.64	2.06	2.4728	2.885	3.297			
0.1927	0.1546	0.1303	0.1132	0.1009			

TABLE 1

```
             0           9           3
ITMAX   =   10          IPRINT =   0          N       =    4
EPS1    =   1.0E-10     EPS2   =   1.0E-02
XOLD(1)...XOLD(4)       10.0       0.0        0.0        0.0
5.0         7.5         10.0
180.0       160.0       140.0      120.0      100.0      80.0       60.0       45.0
35.0
24.3        180.4       0.53       12.15      53.5
   0.12791E-04
1.99        7.125       0.0        0.0        118.61     41.382     0.0
0.239       400.0       -1.0       -1.0       16.0
1.25        0.4         19.6       0.04       0.1303     100.0      27.0       11.5
0.0         0.0         -1.0       0.00238
0.0         5.75        8.51       14.67      16.91      4.65       4.62       2.12
0.78        0.0
0.35        -1.0        -1.0       -1.0
-1.0        -1.0        -1.0       -1.0       -1.0       -1.0       -1.0       -1.0
-1.0        -1.0        -1.0       -1.0       -1.0
-1.0        -1.0        -1.0       -1.0
-1.0        -1.0        -1.0       -1.0       -1.0       -1.0       -1.0       -1.0
-1.0        2.75        -1.0       -1.0       -1.0       -1.0       -1.0       -1.0
0
```

TABLE 2

```
             0             9             3
ITMAX   =    10          IPRINT =   0            N        =     4
EPS1    =   1.0E-10      EPS2   =   1.0E-02
XOLD(1)...XOLD(4)        10.0       0.0          0.0          0.0
5.0          7.5         10.0
180.0        160.0       140.0      120.0        100.0        80.0        60.0        45.0
35.0
-1.0         -1.0        -1.0       -1.0         -1.0         -1.0        -1.0        -1.0
       -0.1E 01
-1.0         -1.0        -1.0       -1.0         -1.0         -1.0        -1.0        -1.0
-1.0         -1.0        -1.0       -1.0         -1.0         -1.0        -1.0        -1.0
-1.0         -1.0        -1.0       -1.0         -1.0         -1.0        -1.0        -1.0
-1.0         -1.0        -1.0       -1.0         -1.0         -1.0        -1.0        -1.0
-1.0         -1.0        -1.0       -1.0         -1.0         -1.0        -1.0        -1.0
-1.0         -1.0        -1.0       -1.0         -1.0         -1.0        -1.0        -1.0
-1.0         -1.0        -1.0       -1.0         -1.0         -1.0        -1.0        -1.0
-1.0         -1.0        -1.0       -1.0         -1.0         -1.0        -1.0        -1.0
-1.0         -1.0        -1.0       -1.0         -1.0         -1.0        -1.0        -1.0
-1.0         -1.0        -1.0       -1.0         -1.0         -1.0        -1.0        -1.0
-1.0         -1.0        -1.0       -1.0         -1.0         -1.0        -1.0        -1.0
-1.0         -1.0        -1.0       -1.0         -1.0         -1.0        -1.0        -1.0
1
```

TABLE 3

8.2 Computer Listing

The main program and all necessary subroutines utilized to compute the mathematical model are listed in this section.

```
C     MAIN PROGRAM
C
      REAL KVA,KVT,KXOLD1
      REAL*8 A,XINC,EPS1,SIMUL,DABS
      DIMENSION AY(15,4),AXOLD(4)
      DIMENSION XIN(11),YIN1(11),YIN2(11),YIN3(11),CPIN(11)
      DIMENSION AE1(44),AE2(44),AE3(44),AEP(44)
      DIMENSION CLIN(11),DLIN(11)
      DIMENSION BIN(21),AIN(21),AEA(80),
     1XOLD(21),XINC(21),A(5,5),Y(15),D(15,4),
     2ROB(11),AEL(40),AEDL(40),KVT(20),DGAMMA(40),C(100)
      DIMENSION ARSP(5),DILN(5),ADIL(16)
      DIMENSION VOLD(4)
C
C     DUCK FIT SECTION
C
      READ (5,300) (XIN(I),I=1,11)
      READ (5,300) (YIN1(I),I=1,11)
      READ (5,300) (YIN2(I),I=1,11)
      READ (5,300) (YIN3(I),I=1,11)
      READ (5,300) (CPIN(I),I=1,11)
      READ (5,300) (BIN(I), I=1,21)
      READ (5,300) (AIN(I), I=1,21)
      READ (5,300) (ROB(I),I=1,11)
      READ (5,300) (CLIN(I),I=1,11)
      READ (5,300) (DLIN(I),I=1,11)
      READ (5,300) (ARSP(I),I=1,5)
      READ (5,300) (DILN(I),I=1,5)
  300 FORMAT (8F10.0)
      CALL UGLYDK (11,2,1,XIN,YIN1,0.0,0.0,AE1)
      CALL UGLYDK (11,2,2,XIN,YIN2,0.0,0.0,AE2)
      CALL UGLYDK (11,2,2,XIN,YIN3,0.0,0.0,AE3)
      CALL UGLYDK (11,2,2,XIN,CPIN,0.0,0.0,AEP)
      CALL UGLYDK (21,2,0,BIN,AIN,0.0,0.0,AEA)
      CALL UGLYDK (11,2,0,ROB,CLIN,0.0,0.0,AEL)
      CALL UGLYDK (11,2,0,ROB,DLIN,0.0,0.0,AEDL)
```

```
      CALL UGLYDK (5,2,0,ARSP,DILN,0.0,0.0,ADIL)
C
C     READ AND PRINT DATA
C
    1 CONTINUE
      READ (5,310) IFAM,NGAM,NVT
  310 FORMAT (8I10)
    2 READ (5,100) ITMAX,IPRINT,N,EPS1,EPS2,(VOLD(I),I=1,N)
      WRITE (6,205) ITMAX,IPRINT,N,EPS1,EPS2,N,(VOLD(I),I=1,N)
      READ (5,300) (KVT(I),I=1,NVT)
      READ (5,300) (DGAMMA(I),I=1,NGAM)
      WRITE (6,315) (KVT(I),I=1,NVT)
  315 FORMAT ('0',9X,'SOLUTIONS ARE COMPUTED FOR THE FOLLOWING TRUE',
     1' WIND SPEEDS (KNOTS)'//(10F10.5/))
      WRITE (6,316) (DGAMMA(I),I=1,NGAM)
  316 FORMAT ('0',9X,'SOLUTIONS ARE COMPUTED FOR THE FOLLOWING TRUE',
     1' WIND ANGLES FOR EACH WIND SPEED'//(10F10.5/))
      VOLD(2)=VOLD(2)*.0174533
C
C     CALL GEOM TO EVALUATE REQUIRED GEOMETRIC COEFICENTS AND CONSTANTS
C
      CALL GEOM (C,ROB,AEL,AEDL,ARSP,ADIL)
C
C     ZERO Y AND D ARRAYS
C
      DO 6 I=1,13
      Y(I)=0.0
      DO 6 J=1,4
      AY(I,J)=0.0
    6 D(I,J)=0.0
      DO 80 II=1,NVT
      DO 15 I=1,4
   15 XOLD(I)=VOLD(I)
      VT=KVT(II)*1.689
      DO 70 III=1,NGAM
      GAMMA=DGAMMA(III)*.0174533
```

```
C
C     NEWTON-RAPHSON ITERATICN
C
    7 CONTINUE
      DO 50 ITER=1,ITMAX
C
C     SET UP AUGMENTED VALUES FOR DIRIVATIVES
C
      DO 8 I=1,4
    8 AXOLD(I)=XOLD(I)+EPS2
C
C     CALL SAIL TO EVALUATE AERCDYNAMIC FORCES, MOMENTS AND PARTIALS
C
      CALL SAIL (XOLD,AXOLD,VT,GAMMA,C,Y,AY,KVA,DBETTA,BIN,AEA)
C
C     CALL RTMOM TO EVALUATE THE HYDROSTATIC RIGHTING MOMENT
C
      CALL RTMOM (XOLD,AXOLD,C,Y,AY)
C
C     CALL HULL TO EVALUATE HULL DRAG
C
      CALL HULL (XOID,AXOLD,C,Y,AY,AE1,AE2,AE3,AEP,XIN)
C
C     CALL KEEL TO EVALUATE INDUCED KEEL FORCES
C
      CALL KEEL (XOLD,AXOLD,C,Y,AY)
C
C     CALL RUDDR TO EVALUATE INDUCED RUDDER FCRCES
C
      CALL RUDDR(XOLD,AXOLD,C,Y,AY)
C
C     COMPUTE PARTIAL DIRIVATIVES OF COMPONENT FORCES AND MOMENTS
C
      DO 10 I=1,15
      DO 10 J=1,4
   10 D(I,J)=(AY(I,J)-Y(I))/EPS2
```

```
C
C     CALL ON CALCN TO SET UP THE A MATRIX
C
      CALL CALCN (A,Y,D)
C
C     CALL SIMUL TO COMPUTE JACOBIAN AND CORRECTION IN XINC
C
      DETER=SIMUL (N,A,XINC,EPS1,1,5)
      IF (DETER.NE.0.0) GO TO 20
      WRITE (6,201)
      GO TO 60
C
C     CHECK FOR CONVERGENCE AND UPDATE XOLD VALUES
C
   20 ITCON=1
      DO 25 I=1,N
      IF (DABS(XINC(I)).GT.EPS2) ITCON=0
   25 XOLD(I)=XOLD(I)+XINC(I)
      IF (IPRINT.EQ.1) WRITE (6,202) ITER,DETER,N,(XOLD(I),I=1,N)
      IF (IPRINT.EQ.1) WRITE (6,206) KVA,DBETTA
      IF (IPRINT.EQ.1) WRITE (6,302) (I,Y(I),I=1,15)
      IF (ITCON.EQ.0) GO TO 50
      WRITE (6,208)
  208 FORMAT ('0',9X,'--------------------------------------------------')
      WRITE (6,207) KVT(II),DGAMMA(III)
  207 FORMAT ('0',9X,'KVT=  ',F10.5,'  KNOTS',10X,'DGAMMA=  ',F10.5,
     1'  DEGREES')
      WRITE (6,206) KVA,DBETTA
  206 FORMAT ('0',9X,'KVA=  ',F10.5,'  KNOTS',10X,'DBETTA=  ',F10.5,
     1'  DEGREES')
      IF (IFAM.EQ.1) WRITE (6,302) (I,Y(I),I=1,15)
  302 FORMAT ('0',9X,'FORCES AND MOMENTS'//(10X,I2,4X,E14.5,10X,I2,
     14X,E14.5,10X,I2,4X,E14.5//))
      KXOLD1=XOLD(1)/1.689
      DXOLD2=XOLD(2)*57.3
      WRITE (6,203) ITER,KXOLD1,DXOLD2,XOLD(3),XOLD(4)
```

```
      WRITE (6,208)
       GO TO 60
   50 CONTINUE
C
      WRITE (6,204)
      WRITE (6,207) KVT(II),DGAMMA(III)
      WRITE (6,206) KVA,DBETTA
      KXOLD1=XOLD(1)/1.689
      DXOLD2=XOLD(2)*57.3
      WRITE (6,203) ITER,KXOLD1,DXOLD2,XOLD(3),XOLD(4)
      GO TO 60
C
C     PROGRAM CONTROL SECTION
C
   60 CONTINUE
   70 CONTINUE
   80 CONTINUE
      READ (5,301) KEND
  301 FORMAT (I1)
      IF (KEND.EQ.0) GO TO 1
      IF (KEND.EQ.1) STOP
C
C     FORMATS FOR INPUT AND OUTPUT STATEMENTS
C
  100 FORMAT ( 10X,I3,17X,I1,19X,I3/ 10X,E7.1,13X,E7.1/ (20X, 5F10.3) )
  201 FORMAT ( 38HOMATRIX IS ILL-CONDITIONED OR SINGULAR )
  202 FORMAT ( 10HOITER    = ,I8/  10H DETER   = , E18.5/
     2   26H               XOLD(1)...XOLD(, I2, 1H) / (1H ,1P4E16.6) )
  203 FORMAT ( 24HOSUCCESSFUL CONVERGENCE / 10HOITER    = , I3//
     110X,'BOAT SPEED=  ',F10.5,'  KNOTS',10X,'HEEL ANGLE=  ',F10.5,
     2'  DEGREES'//10X,'LEEWAY ANGLE=  ',F10.5,'  DEGREES',
     310X,'RUDDER ANGLE=  ',F10.5,'DEGREES')
  204 FORMAT ( 15H NO CONVERGENCE )
  205 FORMAT ( 10H1ITMAX  = , I8/  10H IPRINT = , I8/  10H N      = ,
     1   I8/  10H EPS1   = ,1PE14.1/ 10H EPS2  = , 1PE14.1/
     2   26H0              XOLD(1)...XOLD(, I2, 1H) / 1H / (1H ,1P4E16.6) )
```

```
      STOP
C
      END
```

```
      SUBROUTINE GEOM (C,ROB,AEL,AEDL,ARSP,ADIL)
      DIMENSION C(100),ROB(11),AEL(40),AEDL(40)
      DIMENSION ARSP(5),DILN(5),ADIL(16)
C
C     ZERO C ARRAY
C
      DO 10 I=1,100
   10 C(I)=0.0
C
C     READ REQUIRED CONSTANTS
CC
      READ (5,100) C(2),C(7),C(8),C(9),C(10)
      READ (5,200) C(11)
  200 FORMAT (E14.7)
      READ (5,100) (C(I),I=12,18)
      READ (5,100) C(1),C(3),C(4),C(5),C(6)
      READ (5,100) (C(I),I=19,30)
      READ (5,100) (C(I),I=31,40)
      READ (5,100) (C(I),I=41,44)
      READ (5,100) (C(I),I=45,56),C(61)
      READ (5,100) (C(I),I=57,60)
      READ (5,100) (C(I),I=62,71)
C
C     DEFAULT SYNTHESIS MODEL
C
      CALL SYNTH (C,ROB,AEL,AEDL,ARSP,ADIL)
      STOT=C(72)
C
C     WRITE SECTION
C
      WRITE (6,150) C(2),C(7),C(8),C(9),C(10)
  150 FORMAT ('1',9X,'LWL=  ',F10.5//10X,'DISPLACEMENT TO LENGTH=  ',
     1F10.5//10X,'PRISMATIC=  ',F10.5//10X,'L/H=  ',F10.5//
     210X,'LCB=  ',F10.5)
      WRITE (6,220) (C(I),I=11,18),STOT
  220 FORMAT ('0',9X,'KINEMATIC VISCOSITY=  ',E14.7//10X,
```

```
     1'DENSITY OF WATER=  ',F10.5//10X,'MEAN KEEL CHORD=  ',F10.5//
     210X,'MEAN RUDDER CHORD=  ',F10.5//10X,
     3'ROUGHNESS AND CORRELATION FACTOR=  ',F10.5//10X,
     4'FAIRBODY WETTED SURFACE=  ',F10.5//10X,'KEEL WETTED SURFACE=  ',
     5F10.5//10X,'RUDDER WETTED SURFACE=  ',F10.5//10X,
     6'TOTAL WETTED SURFACE=  ',F10.5)
      WRITE (6,110) C(1),C(3),C(4),C(5),C(6)
  110 FORMAT ('0',9X,'BWL /LWL=  ',F10.5//10X,'SAIL AREA=  ',F10.5//
     110X,'HEELING MOMENT ARM=  ',F10.5//10X,'PITCHING MOMENT ARM=  ',
     2F10.5//10X,'DISTANCE FROM FP TO RUDDER STOCK=  ',F10.5)
  100 FORMAT (8F10.0)
      WRITE (6,230) (C(I),I=19,30)
  230 FORMAT ('0',9X,'CLOSE HAULED LIFT COEF=  ',F10.5//
     110X,'RIG FORM DRAG COEF=  ',F10.5//
     210X,'CROSS SECTIONAL AREA OF RIG=  ',F10.5//
     310X,'SAIL FRICTIONAL COEF=  ',F10.5//
     410X,'INDUCED DRAG COEF / LIFT COEF**2=  ',F10.5//
     510X,'LATERAL AREA OF HULL=  ',F10.5//
     610X,'TRANSVERSE AREA OF HULL=  ',F10.5//
     710X,'DISTANCE BETWEEN CE SAIL AND FP=  ',F10.5//
     810X,'OFF WIND DRAG COEF=  ',F10.5//
     910X,'OFF WIND LIFT COEF=  ',F10.5//
     110X,'DISTANCE BETWEEN CE HULL AND FP=  ',F10.5//
     210X,'DENSITY OF AIR=  ',F10.5)
      WRITE (6,240)(C(I),I=31,40)
  240 FORMAT ('0',9X,'HORIZONTAL DISTANCE FROM MAXIMUM DRAFT OF',
     1' PROFILE MEASUREMENTS'//5F10.5//10X,
     2'PROFILE MEASUREMENTS'//(5F10.5))
      WRITE (6,245)(C(I),I=41,43)
  245 FORMAT ('0',9X,'DRAFT OF CANOE BODY / TOTAL DRAFT=  ',F10.5//
     110X,'HYDRODYNAMIC LIFT COEF=  ',F10.5//
     210X,'LAMDA X LIFT / DRAG=  ',F10.5)
      WRITE (6,250) C(61),(C(I),I=45,56)
  250 FORMAT ('0',9X,'DISTANCE FROM LWL TO VERTICAL HYDRO CE=  ',F10.5//
     110X,'*** RUDDER SECTION ***'//
     210X,'DISTANCE FROM LWL TO ROOT CHORD=  ',F10.5//
```

```
     310X,'ROOT CHORD=  ',F10.5//10X,'TIP CHORD=  ',F10.5//
     410X,'RUDDER SPAN=  ',F10.5//10X,'SWEEP ANGLE=  ',F10.5//
     510X,'ASPECT RATIO=  ',F10.5//10X,'MEAN CHORD=  ',F10.5//
     610X,'DCL / DALPHA=  ',F10.5//10X,'CROSS FLOW DRAG COEF=  ',F10.5//
     710X,'DCM / DCL=  ',F10.5//10X,'FORM DRAG COEF=  ',F10.5//
     810X,'RUDDER AREA=  ',F10.5)
      WRITE (6,254)
  254 FORMAT ('0',9X,'*** RIGHTING MOMENT SECTION ***')
      WRITE (6,255) (C(I),I=57,60)
  255 FORMAT ('0',9X,'METACENTRIC HEIGHT / LWL=  ',F10.5//
     110X,'FROUDE NUMBER RIGHTING MOMENT COEF=  ',E14.7//
     210X,'NON LINEAR HYDROSTATIC COEF=  ',E14.7//
     310X,'DISTANCE BELOW LWL TO CG=  ',F10.5)
      WRITE (6,260) (C(I),I=62,71)
  260 FORMAT ('0',9X,'*** SAIL SECTION ***'//
     110X,'I=  ',F10.5,10X,'J=  ',F10.5//
     210X,'P=  ',F10.5,10X,'E=  ',F10.5//
     310X,'AR FOR TRI=  ',F10.5,10X,'AR MAIN=  ',F10.5//
     410X,'AR SAIL PLAN=  ',F10.5//
     510X,'CE SAIL PLAN AFT MAST=  ',F10.5//
     610X,'FORWARD OVERHANG=  ',F10.5//
     710X,'FREEBOARD=  ',F10.5)
      WRITE (6,300)
  300 FORMAT ('1')
      RETURN
C
      END
```

```
      SUBROUTINE SYNTH (C,ROB,AEL,AEDL,ARSP,ADIL)
      DIMENSION C(100),ROB(11),AEL(40),AEDL(40)
      DIMENSION ARSP(5),DILN(5),ADIL(16)
      DIMENSION ARSPO(1),DILO(1)
      IF (C(2).EQ.-1.0) C(2)=30.0
      IF (C(7).EQ.-1.0) C(7)=225.0
      IF (C(8).EQ.-1.0) C(8)=0.53
      IF (C(1).EQ.-1.0) C(1)=0.3633-0.002018*C(2)
      IF (C(9).EQ.-1.0) C(9)=10975.0*C(1)/C(7)
      IF (C(6).EQ.-1.0) C(6)=0.95*C(2)
      IF (C(10).EQ.-1.0) C(10)=53.0
      IF (C(11).EQ.-1.0) C(11)=0.12791E-04
      IF (C(12).EQ.-1.0) C(12)=1.99
      IF (C(13).EQ.-1.0) C(13)=0.23*C(2)
      IF (C(15).EQ.-1.0) C(15)=0.0
      IF (C(16).EQ.-1.0) C(16)=C(2)*C(2)*(1.7/C(9)+3.5E-05*C(7)*C(9))
     1*(0.0712*(C(1)*C(9)-4.0)+1.0)
      IF (C(19).EQ.-1.0) C(19)=1.25
      IF (C(20).EQ.-1.0) C(20)=0.4
      IF (C(22).EQ.-1.0) C(22)=0.04
      IF (C(3).NE.-1.0) GO TO 30
      C(3)=C(2)*C(2)
   30 CONTINUE
      IF (C(62).NE.-1.0) GO TO 32
      C(62)=SQRT(C(3)/0.4044)
      C(63)=C(62)/3.0
      C(64)=C(62)/1.1
      C(65)=C(64)/3.0
      C(66)=3.0
      C(67)=3.0
      C(68)=C(62)*C(62)/C(3)
   32 CONTINUE
      ARSPO(1)=C(68)
      IF (C(23).EQ.-1.0) CALL EVALDK (5,1,ARSP,ARSPO,DILO,ADIL)
      IF (C(67).LE.C(66)) GO TO 35
      IF (C(23).EQ.-1.0) C(23)=DILO(1)+0.00892*(18.2-C(68))*(C(67)-C(66)
```

```
  1)
      GO TO 40
   35 CONTINUE
      IF (C(23).EQ.-1.0) C(23)=DILO(1)
   40 CONTINUE
      IF (C(27).EQ.-1.0) C(27)=0.0
      IF (C(28).EQ.-1.0) C(28)=0.0
      IF (C(30).EQ.-1.0) C(30)=0.00238
      C(31)=0.0
      IF (C(32).EQ.-1.0) C(32)=0.1*C(2)
      IF (C(33).EQ.-1.0) C(33)=0.2*C(2)
      IF (C(34).EQ.-1.0) C(34)=0.3*C(2)
      IF (C(35).EQ.-1.0) C(35)=0.55*C(2)
      IF (C(36).EQ.-1.0) C(36)=0.146*C(2)+2.0
      IF (C(37).EQ.-1.0) C(37)=0.98*C(36)
      IF (C(38).EQ.-1.0) C(38)=0.95/C(9)*C(2)
      IF (C(39).EQ.-1.0) C(39)=0.7/C(9)*C(2)
      C(40)=0.0
      IF (C(41).EQ.-1.0) C(41)=C(2)/C(9)/C(36)
   42 CALL LAMC (C,ROB,AEL,AEDL)
      IF (C(26).EQ.-1.0) C(26)=C(29)
      IF (C(69).EQ.-1.0) C(69)=0.02*(C(63)+C(65))
      IF (C(70).EQ.-1.0) C(70)=C(63)+C(69)-C(26)
      IF (C(71).EQ.-1.0) C(71)=0.057*C(2)+2.0
      IF (C(24).EQ.-1.0) C(24)=(2.1*C(2)+C(70))/2.0*C(71)
      IF (C(25).EQ.-1.0) C(25)=1.1*C(1)*C(2)*C(71)
      IF (C(4).EQ.-1.0) C(4)=0.43*C(62)+C(71)
      IF (C(17).EQ.-1.0) C(17)=2.1*(C(36)-C(2)/C(9))*C(13)
      IF (C(21).EQ.-1.0) C(21)=C(62)*0.01*(C(64)+C(65))
      IF (C(5).EQ.-1.0) C(5)=0.48*C(62)+C(71)
C
C     RUDDER MODEL
C
      IF (C(6).EQ.-1.0) C(6)=0.95*C(2)
      IF (C(45).EQ.-1.0) C(45)=0.2*C(2)/C(9)
      IF (C(46).EQ.-1.0) C(46)=0.075*C(2)
```

```
      IF  (C(47).EQ.-1.0)  C(47)=0.667*C(46)
      IF  (C(48).EQ.-1.0)  C(48)=0.85*(C(36)-C(45))
      IF  (C(49).EQ.-1.0)   C(49)=0.0
      C(51)=(C(46)+C(47))/2.0
      IF  (C(50).EQ.-1.0)  C(50)=(C(48)+C(48))/C(51)
      IF  (C(52).EQ.-1.0)  C(52)=0.09869*C(50)/(COS(C(49)/57.3)*SQRT(
     1C(50)**2/COS(C(49)/57.3)**4.0+4.0)+1.8)
      IF  (C(53).EQ.-1.0)  C(53)=0.08+0.72*C(47)/C(46)
      IF  (C(54).EQ.-1.0)  C(54)=0.5-(1.11*SQRT(C(50)*C(50)+4.0)+2.0)/
     1(4.0*(C(50)+2.0))
      IF  (C(55).EQ.-1.0)  C(55)=0.0065
      IF  (C(56).EQ.-1.0)  C(56)=C(51)*C(48)
      IF  (C(57).EQ.-1.0)  C(57)=1119.0*C(1)**3.0/C(7)-0.335/C(9)
      IF  (C(58).NE.-1.0)  GO TO 50
      IF  (C(1).GT.0.28)  C(58)=(7.8-56350.0*(C(1)-0.28)**2.75)*1.0E-05
      IF  (C(1).LE.0.28)  C(58)=7.8E-05
   50 CONTINUE
      IF  (C(59).EQ.-1.0)  C(59)= C(57)*(0.0822*C(1)-0.02162)
      IF  (C(60).EQ.-1.0)  C(60)=C(2)*0.025
      IF  (C(61).EQ.-1.0)  C(61)=0.333*C(36)
      IF  (C(14).EQ.-1.0)  C(14)=C(51)
      IF  (C(18).EQ.-1.0)  C(18)=2.1*C(56)
      C(72)=C(16)+C(17)+C(18)
      RETURN
C
      END
```

```
      SUBROUTINE LAMC (C,ROB,AEL,AEDL)
      DIMENSION B3(7),BB3(7),XB3(7),C(50),ROB(11)
      DIMENSION ROBO(1),AEL(40),ATEN(1),ALD(1),AEDL(40)
      COMMON U,Z3,QL,RO
      COMMON XB3,B3
      Z3=C(35)
      RO=C(12)
      U=1.0
      QL=C(2)
      B3(6)=0.0
      DO 10 I=1,5
      XB3(I)=C(30+I)
      B3(I)=C(35+I)
   10 BB3(I)=B3(I)**2
      CALL YN (1.0,DYV,DNV)
      A2=RO*QL*QL/2.0
      A3=A2*QL
      DYV=DYV/A2
      DNV=DNV/A3
      ROBO(1)=C(41)
      CALL EVALDK (11,1,ROB,ROBC,ATEN,AEL)
      CALL EVALDK (11,1,ROB,ROBO,ALD,AEDL)
      IF (C(42).EQ.-1.0) C(42)=ATEN(1)*DYV*(-1.0)
      IF (C(43).EQ.-1.0) C(43)=ALD(1)
      IF (C(29).EQ.-1.0) C(29)=(1.0-DNV/DYV)*C(35)
      RETURN
C
      END
      FUNCTION BD(X)
      DIMENSION XB3(7),B3(7)
      COMMON U,Z3,QL,RO
      COMMON XB3,B3
      DO 80 I=1,4
      J=5-I
      IF (X .GT. XB3(J)) GO TO 90
   80 CONTINUE
```

```
90    BD=B3(J)+(B3(J+1)-B3(J))*(X-XB3(J))/(XB3(J+1)-XB3(J))
      IF (ABS(X).LT.0.0001) DB=B3(1)
      RETURN
      END
      FUNCTION BB(X)
      BB=BD(X)**2
      RETURN
      END
      SUBROUTINE SIMP(QQ,S,N,TGR)
      DIMENSION QQ(31)
      DELTA=S/(N-1)
      TGR=QQ(1)+4.*QQ(N-1)+QC(N)
       NT=(N-3)/2
      DO 10 I=1,NT
      II=2*I
 10   TGR=TGR+4.*QQ(II)+2.*QQ(II+1)
      TGR=TGR*DELTA/3.0
      RETURN
      END
      FUNCTION DBB(X)
      SN=SIGN(1.0,X)
10    DBB=(BB(X+.01*SN)-BB(X))/(.01*SN)
      RETURN
      END
      SUBROUTINE YN (V,QY,QN)
      DIMENSION QC(31),QQQ(31)
      COMMON U,Z3,QL,RO
      DO 30 I=1,31
      QX=(Z3/30.0)*(I-1)
      IF (QX.LT.0.00001) QX=0.00001
      QQ(I)=-U*DBB(QX)*V
   30 QQQ(I)=QX*QQ(I)
      CALL SIMP(QQ,Z3,31,TY3)
      CALL SIMP(QQQ,Z3,31,TN3)
      QY=-3.1416*RO*TY3/2.0
      QN=-3.1416*RO*TN3/2.0
```

```
      RETURN
C
      END
```

```
      SUBROUTINE CALCN (A,Y,D)
      REAL*8 A
      DIMENSION A(5,5),Y(15),D(15,4)
C
C     CLEAR A ARRAY
C
      DO 1 I=1,4
      DO 1 J=1,5
    1 A(I,J)=0.0
C
C     COMPUTE NON-ZERO ELEMENTS OF A
C
      DO 10 I=1,4
      A(1,I)=D(1,I)+D(2,I)+D(3,I)+D(4,I)
      A(2,I)= D(5,I)+D(6,I)+D(7,I)
      A(3,I)=D(10,I)+D(11,I)+D(12,I)+D(13,I)
   10 A(4,I)=D(8,I)+D(9,I)+D(14,I)+D(15,I)
      A(1,5)=-Y(1)-Y(2)-Y(3)-Y(4)
      A(2,5)=-Y(5)-Y(6)-Y(7)
      A(3,5)=-Y(10)-Y(11)-Y(12)-Y(13)
      A(4,5)=-Y(8)-Y(9)-Y(14)-Y(15)
      RETURN
C
      END
```

```
      FUNCTION SIMUL( N,A,X,EPS,INDIC,NRC)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 A,X,EPS,SIMUL
      DIMENSION IROW(50),JCOL(50),JORD(50),Y(50),A(NRC,NRC),X(N)
C
      MAX=N
      IF (INDIC.GE.0) MAX=N+1
C
C     IS N LARGER THAN 50
C
      IF (N.LE.50) GO TO 5
      WRITE (6,200)
      SIMUL=0.0
      RETURN
C
C     BEGIN ELIMINATION PROCEDURE
C
    5 DETER=1.0
      DO 18 K=1,N
      KM1=K-1
C
C     SEARCH FOR THE PIVOT ELEMENT
C
      PIVOT=0.0
      DO 11 I=1,N
      DO 11 J=1,N
C
C     SCAN IROW AND JCOL ARRAYS FOR INVALID PIVOT SUBSCRIPTS
C
      IF (K.EQ.1) GO TO 9
      DO 8 ISCAN=1,KM1
      DO 8 JSCAN=1,KM1
      IF (I.EQ.IROW(ISCAN)) GO TO 11
      IF (J.EQ.JCOL(JSCAN)) GO TO 11
    8 CONTINUE
    9 IF (DABS(A(I,J)).LE.DABS(PIVOT)) GO TO 11
```

```
      PIVOT=A(I,J)
      IROW(K)=I
      JCOL(K)=J
   11 CONTINUE
C
C     INSURE THAT SELECTED PIVOT IS LARGER THAN EPS
C
      IF (DABS(PIVOT).GT.EPS) GC TO 13
      SIMUL=0.0
      RETURN
C
C     UPDATE THE DETERMINANT VALUE
C
   13 IROWK=IROW(K)
      JCOLK=JCOL(K)
      DETER=DETER*PIVOT
C
C     NORMALIZE PIVOT ROW ELEMENTS
C
      DO 14 J=1,MAX
   14 A(IROWK,J)=A(IROWK,J)/PIVCT
C
C     CARRY OUT ELIMINATION AND DEVELOPE INVERSE
C
      A(IROWK,JCOLK)=1.0/PIVOT
      DO 18 I=1,N
      AIJCK=A(I,JCOLK)
      IF (I.EQ.IROWK) GO TO 18
      A(I,JCOLK)=-AIJCK/PIVOT
      DO 17 J=1,MAX
   17 IF (J.NE.JCCLK) A(I,J)=A(I,J)-AIJCK*A(IROWK,J)
   18 CONTINUE
C
C     ORDER SOLUTION VALUES (IF ANY) AND CREATE JORD ARRAY
C
      DO 20 I=1,N
```

```
      IROWI=IROW(I)
      JCOLI=JCOL(I)
      JORD(IROWI)=JCOLI
   20 IF (INDIC.GE.0) X(JCOLI)=A(IROWI,MAX)
C
C     ADJUST SIGN OF DETERMINANT
C
      INTCH=0
      NM1=N-1
      DO 22 I=1,NM1
      IP1=I+1
      DO 22 J=IP1,N
      IF (JORD(J).GE.JORD(I)) GC TO 22
      JTEMP=JORD(J)
      JORD(J)=JORD(I)
      JORD(I)=JTEMP
      INTCH=INTCH+1
   22 CONTINUE
      IF (INTCH/2*2.NE.INTCH) DETER=-DETER
C
C     IF INDIC IS POSITIVE RETURN WITH RESULTS
C
      IF (INDIC.LE.0) GO TO 26
      SIMUL=DETER
      RETURN
C
C     IF INDIC IS NEGATIVE OR ZERO, UNSCRAMBLE THE INVERSE
C     FIRST BY ROWS
C
   26 DO 28 J=1,N
      DO 27 I=1,N
      IROWI=IROW(I)
      JCOLI=JCOL(I)
   27 Y(JCOLI)=A(IROWI,J)
      DO 28 I=1,N
   28 A(I,J)=Y(I)
```

```
C
C     THEN BY COLUMNS
C
      DO 30 I=1,N
      DO 29 J=1,N
      IROWJ=IROW(J)
      JCOLJ=JCOL(J)
   29 Y(IROWJ)=A(I,JCOLJ)
      DO 30 J=1,N
   30 A(I,J)=Y(J)
C
C     RETURN FOR INDIC NEGATIVE OR ZERO
C
      SIMUL=DETER
      RETURN
C
C     FORMAT FOR OUTPUT STATEMENT
C
  200 FORMAT ( 10HON TOO BIG )
C
      END
```

```
      SUBROUTINE UGLYDK(NIN,NCL,NCR,XIN,YIN,ESL,ESR,AE)
C     APRIL 1975 DUCK SRIES    J.E.KERWIN
      DIMENSION XIN(1),YIN(1),AE(1),H(30),D(30),A(900),S(30)
      DATA HALF/0.5E00/,TWO/2.0E00/,SIX/6.0E00/,RAD/1.745329E-02/       PBS10355
      NM1=NIN-1                                                         PBS10356
      NM2=NM1-1                                                         PBS10357
      NM3=NM2-1
      NEQ=NM2                                                           PBS10358
      DO 1 N=1,NM1                                                      PBS10359
      H(N)=XIN(N+1)-XIN(N)                                              PBS10360
    1 D(N)=(YIN(N+1)-YIN(N))/H(N)                                       PBS10361
      IF(NCL.EQ.2) NEQ=NEQ+1
      IF(NCR.EQ.2) NEQ=NEQ+1
      NSQ=NEQ**2
      DO 4 N=1,NSQ                                                      PBS10367
    4 A(N)=0.0                                                          PBS10368
      J=1                                                               PBS10369
      L=1                                                               PBS10370
      IF(NCL.LT.2) GO TO 6
      A(1)=TWO*H(1)                                                     PBS10372
      A(2)=H(1)                                                         PBS10373
      SLP=ESL*RAD                                                       PBS10374
      S(1)=(D(1)-TAN(SLP))*SIX
      J=J+1                                                             PBS10376
      L=L+NEQ+1
      A(L-1)=H(1)
    6 DO 5 N=1,NM2                                                      PBS10380
      IF(N.GT.1) A(L-1)=H(N)                                            PBS10381
      A(L)=TWO*(H(N)+H(N+1))                                            PBS10382
      IF(N.LT.NM2) A(L+1)=H(N+1)                                        PBS10383
      IF(N.EQ.2.AND.NCL.EQ.1) A(L-1)=A(L-1)-H(N)**2/H(N+1)
      IF(N.EQ.1.AND.NCL.EQ.1) A(L)=A(L)+(1.0+H(N)/H(N+1))*H(N)
      IF(N.EQ.NM2.AND.NCR.EQ.1) A(L)=A(L)+(1.0+H(N+1)/H(N))*H(N+1)
      IF(N.EQ.NM3.AND.NCR.EQ.1) A(L+1)=A(L+1)-H(N+2)**2/H(N+1)
      S(J)=(D(N+1)-D(N))*SIX                                            PBS10384
      J=J+1                                                             PBS10385
```

```
5     L=L+NEQ+1                                          PBS10386
      IF(NCR.LT.2) GO TO 7
      A(L-1)=H(NM1)                                      PBS10388
      A(L)=-TWO*H(NM1)                                   PBS10389
      L=L-NEQ                                            PBS10390
      A(L)=-H(NM1)                                       PBS10391
      SLP=ESR*RAD                                        PBS10392
      S(J)=(D(NM1)+TAN(SLP))*SIX
7     CALL SIMQ(A,S,NEQ,KERROR)
      HOLD=S(NEQ)
      IF(NCL.EQ.2) GO TO 8
      DO 9 N=1,NM2                                       PBS10396
      M=NM2-N+2                                          PBS10397
9     S(M)=S(M-1)                                        PBS10398
      IF(NCL.EQ.0) S(1)=0.0
      BUG=H(1)/H(2)
      IF(NCL.EQ.1) S(1)=(1.0+BUG)*S(2)-BUG*S(3)
8     IF(NCR.EQ.0) S(NIN)=0.0
      BUG=H(NM1)/H(NM2)
      IF(NCR.EQ.1) S(NIN)=(1.0+BUG)*S(NM1)-BUG*S(NM2)
      IF(NCR.EQ.2) S(NIN)=HOLD
      DO 10 N=1,NM1                                      PBS10401
      AE(N)=(S(N+1)-S(N))/(SIX*H(N))                     PBS10402
      M=N+NM1
      AE(M)=HALF*S(N)
      M=M+NM1
      AE(M)=D(N)-H(N)*(TWO*S(N)+S(N+1))/SIX
      M=M+NM1
10    AE(M)=YIN(N)
      RETURN
      END
```

```
      SUBROUTINE SIMQ(A,B,N,KS)                                    SIMQ 490
      DIMENSION A(1),B(1)
      TOL=0.0                                                      SIMQ 540
      KS=0                                                         SIMQ 550
      JJ=-N                                                        SIMQ 560
      DO 65 J=1,N                                                  SIMQ 570
      JY=J+1                                                       SIMQ 580
      JJ=JJ+N+1                                                    SIMQ 590
      BIGA=0                                                       SIMQ 600
      IT=JJ-J                                                      SIMQ 610
      DO 30 I=J,N                                                  SIMQ 620
      IJ=IT+I                                                      SIMQ 660
      IF(ABS(BIGA)-ABS(A(IJ))) 20,30,30
   20 BIGA=A(IJ)                                                   SIMQ 680
      IMAX=I                                                       SIMQ 690
   30 CONTINUE                                                     SIMQ 700
      IF(ABS(BIGA)-TOL) 35,35, 40
   35 KS=1                                                         SIMQ 750
      RETURN                                                       SIMQ 760
   40 I1=J+N*(J-2)                                                 SIMQ 800
      IT=IMAX-J                                                    SIMQ 810
      DO 50 K=J,N                                                  SIMQ 820
      I1=I1+N                                                      SIMQ 830
      I2=I1+IT                                                     SIMQ 840
      SAVE=A(I1)                                                   SIMQ 850
      A(I1)=A(I2)                                                  SIMQ 860
      A(I2)=SAVE                                                   SIMQ 870
   50 A(I1)=A(I1)/BIGA                                             SIMQ 910
      SAVE=B(IMAX)                                                 SIMQ 920
      B(IMAX)=B(J)                                                 SIMQ 930
      B(J)=SAVE/BIGA                                               SIMQ 940
      IF(J-N) 55,70,55                                             SIMQ 980
   55 IQS=N*(J-1)                                                  SIMQ 990
      DO 65 IX=JY,N                                                SIMQ1000
      IXJ=IQS+IX                                                   SIMQ1010
      IT=J-IX                                                      SIMQ1020
```

```
      DO 60 JX=JY,N                                                   SIMQ1030
      IXJX=N*(JX-1)+IX                                                SIMQ1040
      JJX=IXJX+IT                                                     SIMQ1050
   60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))                                 SIMQ1060
   65 B(IX)=B(IX)-(B(J)*A(IXJ))                                       SIMQ1070
   70 NY=N-1                                                          SIMQ1110
      IT=N*N                                                          SIMQ1120
      DO 80 J=1,NY                                                    SIMQ1130
      IA=IT-J                                                         SIMQ1140
      IB=N-J                                                          SIMQ1150
      IC=N                                                            SIMQ1160
      DO 80 K=1,J                                                     SIMQ1170
      B(IB)=B(IB)-A(IA)*B(IC)                                         SIMQ1180
      IA=IA-N                                                         SIMQ1190
   80 IC=IC-1                                                         SIMQ1200
      RETURN                                                          SIMQ1210
      END
```

```
      SUBROUTINE EVALDK(NIN,NOUT,XIN,XOUT,YOUT,A)
C     APRIL 1975 SPLINE PROGRAM SERIES   J.E.KERWIN
      DIMENSION XIN(1),XOUT(1),YOUT(1),A(1)
      NM1=NIN-1
      MOUT=IABS(NOUT)
      IF(NOUT.GT.0) GO TO 1
      DEL=(XIN(NIN)-XIN(1))/(MOUT-1)
      DO 2 N=1,MOUT
    2 XOUT(N)=XIN(1)+(N-1)*DEL
    1 J=1
      DO 3 N=1,MOUT
      IF(XOUT(N).GE.XIN(2)) GO TO 4
      J=1
      GO TO 5
    4 IF(XOUT(N).LT.XIN(NM1)) GC TO 6
      J=NM1
      GO TO 5
    6 IF(XOUT(N).GE.XIN(J+1)) GO TO 7
    9 IF (XOUT(N).LT.XIN(J)) GC TO 8
    5 H1=XOUT(N)-XIN(J)
      H2=H1**2
      H3=H1*H2
      J2=J+NM1
      J3=J2+NM1
      J4=J3+NM1
      YOUT(N)=A(J)*H3+A(J2)*H2+A(J3)*H1+A(J4)
      GO TO 3
    7 J=J+1
      GO TO 6
    8 J=J-1
      GO TO 9
    3 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SAIL (XOLD,AXOLD,VT,GAMMA,C,Y,AY,KVA,DBETTA,BIN,AEA)
      REAL KVA,LIFT,LIFT2
      DIMENSION XOLD(4),AXOLD(4),C(100),Y(15),AY(15,4),BIN(21),AEA(80)
      DIMENSION ALPHA(2),BOUT(2)
C
C     COMPUTE APPARENT WIND, VA, AND APPARENT WIND ANGLE, BETTA
C
      PI=3.141593
      ICT=0
      VT1=0.55*VT
    1 CONTINUE
      VTCG=VT1*COS(GAMMA)
      VTSG=VT1*SIN(GAMMA)
      VASQ=(XOLD(1)+VTCG)**2+VTSG**2
      AVASQ=(AXOLD(1)+VTCG)**2+VTSG**2
      BETTA=ABS(ATAN2(VTSG,(XOLD(1)+VTCG)))
      ABETTA=ABS(ATAN2(VTSG,(AXOLD(1)+VTCG)))
      IF (ICT.EQ.1) GO TO 10
C
C     HULL AERO FORCES
C
      BOUT(1)= BETTA
      BOUT(2)=ABETTA
      CALL EVALDK (21,2,BIN,BOUT,ALPHA,AEA)
      FH=0.001428*VASQ*(C(24)*SIN(BETTA)**2+0.3*C(25)*COS(BETTA)**2)
     1/COS(ALPHA(1)-BETTA)
      AFH=0.001428*AVASQ*(C(24)*SIN(ABETTA)**2+0.3*C(25)*COS(ABETTA)**2)
     1/COS(ALPHA(2)-ABETTA)
      YMH=FH*C(2)*(0.2+0.17819*BETTA)*SIN(ALPHA(1))
      AYMH=AFH*C(2)*(0.2+0.17819*ABETTA)*SIN(ALPHA(2))
      ICT=1
      VT1=VT
      GO TO 1
C
C     SAIL FORCES
C
```

```
 10 CONTINUE
    KVA=SQRT(VASQ)/1.689
    DBETTA=BETTA*57.2958
    BETTAO=BETTA
    ABETAO=ABETTA
    VASQO=VASQ
    AVASQO=AVASQ
    VASQ=VASQO*((COS(XOLD(2))*SIN(BETTAO))**2+COS(BETTAO)**2)
    AVASQ=AVASQO*((COS(XOLD(2))*SIN(ABETAO))**2+COS(ABETAO)**2)
    VASQ2=VASQO*((COS(AXOLD(2))*SIN(BETTAO))**2+COS(BETTAO)**2)
    BETTA=ATAN2(COS(XOLD(2))*SIN(BETTAO),COS(BETTAO))
    ABETTA=ATAN2(COS(XOLD(2))*SIN(ABETAO),CCS(ABETAO))
    BETTA2=ATAN2(COS(AXOLD(2))*SIN(BETTAO),COS(BETTAO))
    Q=C(30)/2.0*VASQ
    AQ=C(30)/2.0*AVASQ
    Q2=C(30)/2.0*VASQ2
    LIFT=C(19)*Q*C(3)
    ALIFT=C(19)*AQ*C(3)
    LIFT2=C(19)*Q2*C(3)
    DRAG=(C(20)*C(21)+C(22)*C(3))*Q+C(23)*LIFT**2
   1/Q/C(3)
    ADRAG=(C(20)*C(21)+C(22)*C(3))*AQ+C(23)*ALIFT**2/AQ/C(3)
    DRAG2=(C(20)*C(21)+C(22)*C(3))*Q2+C(23)*LIFT2**2
   1/Q2/C(3)
    IF (BETTA.LE.1.57) GO TO 20
    IF(C(28).NE.0.0) GO TO 11
    LIFT=LIFT*COS(BETTA-1.57)
    ALIFT=ALIFT*COS(ABETTA-1.57)
    LIFT2=LIFT2*COS(BETTA2-1.57)
    GO TO 15
 11 LIFT=LIFT/C(19)*C(28)
    ALIFT=ALIFT/C(19)*C(28)
    LIFT2=LIFT2/C(19)*C(28)
 15 CONTINUE
    IF (C(28).NE.0.0) GO TO 17
    DRAG=DRAG+(1.2*Q*C(3)-DRAG)*SIN(BETTA-1.57)
```

```
      ADRAG=ADRAG+(1.2*AQ*C(3)-ADRAG)*SIN(ABETTA-1.57)
      DRAG2=DRAG2+(1.2*Q2*C(3)-DRAG2)*SIN(BETTA2-1.57)
      GO TO 20
   17 DRAG=C(27)*Q*C(3)
      ADRAG=C(27)*AQ*C(3)
      DRAG2=C(27)*Q2*C(3)
   20 CONTINUE
C
C     COMPUTE AERO DRIVING FCRCES
C
      Y(1)=(LIFT*SIN(BETTA)-DRAG*COS(BETTA))
     1-FH*COS(ALPHA(1))
      AY(1,1)=(ALIFT*SIN(ABETTA)-ADRAG*COS(ABETTA))
     1-AFH*COS(ALPHA(2))
      AY(1,2)=LIFT2*SIN(BETTA2)-DRAG2*COS(BETTA2)
     1-FH*COS(ALPHA(1))
      AY(1,3)=Y(1)
      AY(1,4)=Y(1)
C
C     COMPUTE AERO SIDE FORCE, Y(5)
C
      Y(5)=(LIFT*COS(BETTA)+DRAG*SIN(BETTA))
     1*COS(XOLD(2))+FH*SIN(ALPHA(1))
      AY(5,1)=(ALIFT*COS(ABETTA)+ADRAG*SIN(ABETTA))
     1*COS(XOLD(2))+AFH*SIN(ALPHA(2))
      AY(5,2)=(LIFT2*COS(BETTA2)+DRAG2*SIN(BETTA2))
     1*COS(AXOLD(2))+FH*SIN(ALPHA(1))
      AY(5,3)=Y(5)
      AY(5,4)=Y(5)
C
C     COMPUTE AERO HEELING MCMENT, Y(8)
C     C(4)=DISTANCE BETWEEN CE SAIL AND CE HULL
C
      Y(8)=(LIFT*COS(BETTA)+DRAG*SIN(BETTA))
     1*C(4)
      AY(8,1)=(ALIFT*COS(ABETTA)+ADRAG*SIN(ABETTA))
```

```
     1*C(4)
      AY(8,2)=(LIFT2*COS(BETTA2)+DRAG2*SIN(BETTA2))
     1*C(4)
      AY(8,3)=Y(8)
      AY(8,4)=Y(8)
C
C     COMPUTE YAWING MOMENT DUE TO EARO SIDE FORCES
C
      Y(10)=(Y(5)-FH*SIN(ALPHA(1)))*C(26)+YMH
      AY(10,1)=(AY(5,1)-AFH*SIN(ALPHA(2)))*C(26)+AYMH
      AY(10,2)=(AY(5,2)-FH*SIN(ALPHA(1)))*C(26)+YMH
      AY(10,3)=Y(10)
      AY(10,4)=Y(10)
C
C     COMPUTE SAIL YAWING MOMENT, Y(11)
C     C(5)= DISTANCE FROMCE SAIL TO CENTER OF RESISTANCE
C
      Y(11)=C(5)*Y(1)*SIN(XOLD(2))
      AY(11,1)=C(5)*AY(1,1)*SIN(XOLD(2))
      AY(11,2)=C(5)*AY(1,2)*SIN(AXOLD(2))
      AY(11,3)=Y(11)
      AY(11,4)=Y(11)
      RETURN
C
      END
```

```
      SUBROUTINE HULL (XOLD,AXOLD,C,Y,AY,AE1,AE2,AE3,AEP,XIN)
      DIMENSION XOLD(4),AXOLD(4),C(100),Y(15),AY(15,4)
      DIMENSION AE1(44),AE2(44),AE3(44),AEP(44),XIN(11)
      DIMENSION SLR(2),F1(2),F2(2),F3(2),CPOP(2)
      DIMENSION CFCP(2),CFLH(2),CFLCB(2),DWAVE(2),REFB(2),REKL(2)
      DIMENSION RERD(2),VS(2)
      DIMENSION CFFB(2),CFKL(2),CFRD(2),DFRIC(2)
      SLR(1)=XOLD(1)/1.689/C(2)**0.5
      SLR(2)=AXOLD(1)/1.689/C(2)**0.5
      CALL EVALDK (11,2,XIN,SLR,F1,AE1)
      CALL EVALDK (11,2,XIN,SLR,F2,AE2)
      CALL EVALDK (11,2,XIN,SLR,F3,AE3)
      CALL EVALDK (11,2,XIN,SLR,CPOP,AEP)
      VS(1)=XOLD(1)
      VS(2)=AXOLD(1)
C
C     COMPUTE WAVE RESISTANCE
C
      DO 2 I=1,2
      CFCP(I)=F2(I)*(ABS((C(8)-CPOP(I))*100.0))**1.7
      CFLH(I)=-F3(I)*(16.0-C(9))
      CFLCB(I)=SIN(1.57*SLR(I))*(ABS(62.5-17.5*C(8)-C(10)))**3/100.0
      DWAVE(I)=F1(I)*(C(2)/100.0)**3*C(7)*C(7)*(1.0+CFCP(I)+CFLH(I)
     1+CFLCB(I))
C
C     COMPUTE FRICTIONAL RESISTANCE
C
      REFB(I)=C(2)*VS(I)/C(11)
      REKL(I)=C(13)*VS(I)/C(11)
      RERD(I)=C(14)*VS(I)/C(11)
      IF (REFB(I).LE.0.0) REFB(I)=1.0
      IF (REKL(I).LE.0.0) REKL(I)=1.0
      IF (RERD(I).LE.0.0) RERD(I)=1.0
      CFFB(I)=0.075/(ALOG10(REFB(I))-2.0)**2+C(15)
      CFKL(I)=0.075/(ALOG10(REKL(I))-2.0)**2
      CFRD(I)=0.075/(ALOG10(RERD(I))-2.0)**2
```

```
      DFRIC(I)=C(12)/2.0*VS(I)**2*(CFFB(I)*C(16)+CFKL(I)*C(17)+CFRD(I)
     1*C(18))
    2 CONTINUE
      Y(2)=-ABS((DWAVE(1)+DFRIC(1))/COS(XOLD(2)))
C
C     COMPUTE AUGMENTED FORCES
C
      AY(2,1)=-ABS((DWAVE(2)+DFRIC(2))/COS(XOLD(2)))
      AY(2,2)=-ABS((DWAVE(1)+DFRIC(1))/COS(AXOLD(2)))
      AY(2,3)=Y(2)
      AY(2,4)=Y(2)
      RETURN
C
      END
```

```
      SUBROUTINE RUDDR (XOLD,AXOLD,C,Y,AY)
      DIMENSION XOLD(4),AXOLD(4),C(100),Y(15),AY(15,4)
      A=XOLD(4)/57.3
      AA=AXOLD(4)/57.3
      X3=XOLD(3)/57.3
      XX3=AXOLD(3)/57.3
      CL=C(52)*XOLD(4)+C(53)/C(50)*A**2
      CL4=C(52)*AXOLD(4)+C(53)/C(50)*A**2
      CD=C(55)+CL**2/(2.827*C(50))+0.0166*CL**2
      CD4=C(55)+CL4**2/(2.827*C(50))+0.0166*CL4**2
      Q=0.5*C(12)*XOLD(1)**2*C(56)
      AQ=0.5*C(12)*AXOLD(1)**2*C(56)
C
C     COMPUTE RUDDER SIDE FORCE,Y(7)
C
      FAC=CL*COS(X3)-CD*SIN(X3)
      Y(7)=-FAC*Q*COS(XOLD(2))
      AY(7,1)=-FAC*AQ*COS(XOLD(2))
      AY(7,2)=-FAC*Q*COS(AXOLD(2))
      AY(7,3)=-(CL*COS(XX3)-CD*SIN(XX3))*Q*COS(XOLD(2))
      AY(7,4)=-(CL4*COS(X3)-CD4*SIN(X3))*Q*COS(XOLD(2))
C
C     COMPUTE RUDDER DRAG, Y(4)
C
      FAC=CL*SIN(X3)+CD*COS(X3)
      Y(4)=-FAC*Q
      AY(4,1)=-FAC*AQ
      AY(4,2)=Y(4)
      AY(4,3)=-(CL*SIN(XX3)+CD*COS(XX3))*Q
      AY(4,4)=-(CL4*SIN(X3)+CD4*COS(X3))*Q
C
C     COMPUTE YAWING MOMENT FROM RUDDER, Y(13)
C
      CN=CL*COS(A)+CD*SIN(A)
      CN4=CL4*COS(AA)+CD4*SIN(AA)
      CM=(0.25-C(54))*C(52)-0.5*C(53)/C(50)*A**2
```

```
      CM4=(0.25-C(54))*C(52)-0.5*C(53)/C(50)*AA**2
      IF (CN.LT.0.0001) CN=0.0001
      CP=-CM/CN*C(51)
      IF (CN4.LT.0.0001) CN4=0.0001
      CP4=-CM4/CN4*C(51)
      Y(13)=Y(7)*(C(6)+CP)
      AY(13,1)=AY(7,1)*(C(6)+CP)
      AY(13,2)=AY(7,2)*(C(6)+CP)
      AY(13,3)=AY(7,3)*(C(6)+CP)
      AY(13,4)=AY(7,4)*(C(6)+CP4)
C
C     COMPUTE HEELING MOMENT FROM RUDDER
C
      CPS=(0.4244*CL*COS(A)+CD*SIN(A))/CN*C(48)
      CPS4=(0.4244*CL4*COS(AA)+CD4*SIN(AA))/CN4*C(48)
      Y(15)=CL*Q*(C(45)+CPS)
      AY(15,1)=CL*AQ*(C(45)+CPS)
      AY(15,2)=Y(15)
      AY(15,3)=Y(15)
      AY(15,4)=CL4*Q*(C(45)+CPS)
      RETURN
C
      END
```

```
      SUBROUTINE KEEL (XOLD,AXOLD,C,Y,AY)
      DIMENSION XCLD(4),AXOLD(4),C(100),Y(15),AY(15,4)
      Q=C(12)*(XOLD(1)*COS(XOLD(2)))**2/2.0
      Q1=C(12)*(AXOLD(1)*COS(XOLD(2)))**2/2.0
      Q2=C(12)*(XOLD(1)*COS(AXOLD(2)))**2/2.0
      XL=C(42)*Q*XOLD(3)/57.3*C(2)**2
      XL1=C(42)*Q1*XOLD(3)/57.3*C(2)**2
      XL2=C(42)*Q2*XOLD(3)/57.3*C(2)**2
      XL3=C(42)*Q*AXOLD(3)/57.3*C(2)**2
C
C     COMPUTE KEEL SIDE FORCE, Y(6)
C
      Y(6)=-XL
      AY(6,1)=-XL1
      AY(6,2)=-XL2
      AY(6,3)=-XL3
      AY(6,4)=Y(6)
C
C     COMPUTE INDUCED KEEL DRAG , Y(3)
C
      Y(3)=-XL*XOLD(3)/57.3/C(43)/COS(XOLD(2))**2
      AY(3,1)=-XL1*XOLD(3)/57.3/C(43)/COS(XOLD(2))**2
      AY(3,2)=-XL2*XOLD(3)/57.3/C(43)/COS(AXOLD(2))**2
      AY(3,3)=-XL3*AXOLD(3)/57.3/C(43)/COS(XOLD(2))**2
      AY(3,4)=Y(3)
C
C     COMPUTE MOMENT DUE TO KEEL SIDE FORCE, Y(12)
C
      Y(12)=C(29)*Y(6)
      AY(12,1)=C(29)*AY(6,1)
      AY(12,2)=C(29)*AY(6,2)
      AY(12,3)=C(29)*AY(6,3)
      AY(12,4)=Y(12)
C
C     COMPUTE HEELING MOMENT
C
```

```
      Y(14)=C(61)*Y(6)/COS(XOLD(2))*(-1.0)
      AY(14,1)=C(61)*AY(6,1)/CCS(XOLD(2))*(-1.0)
      AY(14,2)=C(61)*AY(6,2)/COS(AXOLD(2))*(-1.0)
      AY(14,3)=C(61)*AY(6,3)/COS(XOLD(2))*(-1.0)
      AY(14,4)=Y(14)
      RETURN
C
      END
```

```
      SUBROUTINE RTMOM (XOLD,AXOLD,C,Y,AY)
      DIMENSION XOLD(4),AXOLD(4),C(100),Y(15),AY(15,4)
C
C     COMPUTE HYDROSTATIC RIGHTING MOMENT
C
C     C(1)= BWL / LWL
C     C(2)= LWL
      FAC=-0.00224*C(7)*C(2)**4.0
      FN=XOLD(1)/SQRT(32.2*C(2))
      FN1=AXOLD(1)/SQRT(32.2*C(2))
      Y(9)=FAC*XOLD(2)*(C(57)+C(58)*FN+C(59)*XOLD(2))
     1+C(60)*SIN(XOLD(2)))
C
C     COMPUTE AUGMENTED FORCES
C
      AY(9,1)=FAC*XOLD(2)*(C(57)+C(58)*FN1+C(59)*XOLD(2))
     1+C(60)*SIN(XOLD(2)))
      AY(9,2)=FAC*AXOLD(2)*(C(57)+C(58)*FN+C(59)*AXOLD(2))
     1+C(60)*SIN(AXOLD(2)))
      AY(9,3)=Y(9)
      AY(9,4)=Y(9)
      RETURN
C
      END
```

8.3 Example Output

Two example computer outputs are listed in this section. The first is the yacht Antiope and the second is the model generated sailing boat which is produced if all geometric parameters are defaulted.

```
ITMAX  =        10
IPRINT =         0
N      =         4
EPS1   =         1.0D-10
EPS2   = ,       1.0E-02
         XOLD(1)...XOLD( 4)

    1.000000E 01    0.000000E-01    0.000000E-01    0.000000E-01
         SOLUTIONS ARE COMPUTED FOR THE FOLLOWING TRUE WIND SPEEDS (KNOTS)

  5.00000   7.50000  10.00000
         SOLUTIONS ARE COMPUTED FOR THE FOLLOWING TRUE WIND ANGLES FOR EACH WIND SPEED

180.00000 160.00000 140.00000 120.00000 100.00000  80.00000  60.00000  45.00000  35.00000
```

LWL= 24.30000

DISPLACEMENT TO LENGTH= 180.39990

PRISMATIC= 0.53000

L/H= 12.15000

LCB= 53.50000

KINEMATIC VISCOSITY= 0.1279100E-04

DENSITY OF WATER= 1.99000

MEAN KEEL CHORD= 7.12500

MEAN RUDDER CHORD= 0.00000

ROUGHNESS AND CORRELATION FACTOR= 0.00000

FAIRBODY WETTED SURFACE= 118.61000

KEEL WETTED SURFACE= 41.38200

RUDDER WETTED SURFACE= 0.00000

TOTAL WETTED SURFACE= 159.99200

BWL /LWL= 0.23900

SAIL AREA= 400.00000

HEELING MOMENT ARM= 16.27361

PITCHING MOMENT ARM= 17.84613

DISTANCE FROM FP TO RUDDER STOCK= 16.00000

CLOSE HAULED LIFT COEF= 1.25000

RIG FORM DRAG COEF= 0.40000

CROSS SECTIONAL AREA OF RIG= 19.60001

SAIL FRICTIONAL COEF= 0.04000

INDUCED DRAG COEF / LIFT COEF**2= 0.13030

LATERAL AREA OF HULL= 100.00000

TRANSVERSE AREA OF HULL= 27.00000

DISTANCE BETWEEN CE SAIL AND FP= 11.50000

OFF WIND DRAG COEF= 0.00000

```
OFF WIND LIFT COEF=     0.00000
DISTANCE BETWEEN CE HULL AND FP=    11.35764
DENSITY OF AIR=     0.00238
HORIZONTAL DISTANCE FROM MAXIMUM DRAFT OF PROFILE MEASUREMENTS
0.00000   5.75000   8.51000  14.67000  16.91000
PROFILE MEASUREMENTS
4.65000   4.62000   2.12000   0.78000   0.00000
DRAFT OF CANOE BODY / TOTAL DRAFT=     0.35000
HYDRODYNAMIC LIFT COEF=     0.09805
LAMDA X LIFT / DRAG=     2.06498
DISTANCE FROM LWL TO VERTICAL HYDRO CE=     1.54845
*** RUDDER SECTION ***
DISTANCE FROM LWL TO ROOT CHORD=     0.40000
ROOT CHORD=     1.82250
TIP CHORD=     1.21561
RUDDER SPAN=     3.61250
SWEEP ANGLE=     0.00000
ASPECT RATIO=     4.75625
MEAN CHORD=     1.51905
DCL / DALPHA=     0.06745
CROSS FLOW DRAG COEF=     0.56024
DCM / DCL=     0.21407
FORM DRAG COEF=     0.00650
RUDDER AREA=     5.48758
*** RIGHTING MOMENT SECTION ***
METACENTRIC HEIGHT / LWL=     0.05711
FROUDE NUMBER RIGHTING MOMENT COEF=   0.7800000E-04
```

```
NON LINEAR HYDROSTATIC COEF=  -0.1127451E-03

DISTANCE BELOW LWL TO CG=     0.60750

*** SAIL SECTION ***

I=    31.45027          J=     10.48342

P=    28.59114          E=      9.53038

AR FOR TRI=     3.00000           AR MAIN=     3.00000

AR SAIL PLAN=     2.47280

CE SAIL PLAN AFT MAST=     0.40028

FORWARD OVERHANG=    -0.61630

FREEBOARD=     2.75000
```

```
        ----------------------------------------------
        KVT=      5.00000   KNOTS          DGAMMA=   180.00000  DEGREES
        KVA=      2.48617   KNOTS          DBETTA=   179.99980  DEGREES
SUCCESSFUL CONVERGENCE
ITER    =   5
        BOAT SPEED=     2.51383  KNOTS           HEEL ANGLE=     0.00128  DEGREES
        LEEWAY ANGLE=    -0.00390  DEGREES          RUDDER ANGLE=     0.01191DEGREES
        ----------------------------------------------
        ----------------------------------------------
        KVT=      5.00000   KNOTS          DGAMMA=   160.00000  DEGREES
        KVA=      2.60820   KNOTS          DBETTA=   139.03030  DEGREES
SUCCESSFUL CONVERGENCE
ITER    =   2
        BOAT SPEED=     2.72703  KNOTS           HEEL ANGLE=     0.02578  DEGREES
        LEEWAY ANGLE=     0.02088  DEGREES          RUDDER ANGLE=     0.01149DEGREES
        ----------------------------------------------
NO CONVERGENCE
        KVT=      5.00000   KNOTS          DGAMMA=   140.00000  DEGREES
        KVA=      3.24689   KNOTS          DBETTA=    98.17024  DEGREES
SUCCESSFUL CONVERGENCE
ITER    =  11
        BOAT SPEED=     3.36880  KNOTS           HEEL ANGLE=     0.40965  DEGREES
        LEEWAY ANGLE=     0.14458  DEGREES          RUDDER ANGLE=     0.00311DEGREES
        ----------------------------------------------
        KVT=      5.00000   KNOTS          DGAMMA=   120.00000  DEGREES
        KVA=      4.67690   KNOTS          DBETTA=    67.79747  DEGREES
SUCCESSFUL CONVERGENCE
```

```
ITER  =  4
         BOAT SPEED=     4.26724  KNOTS          HEEL ANGLE=     2.74890  DEGREES
         LEEWAY ANGLE=     0.41392  DEGREES          RUDDER ANGLE=     0.19046DEGREES
         ------------------------------------------------
         ------------------------------------------------
         KVT=      5.00000   KNOTS          DGAMMA=   100.00000  DEGREES
         KVA=      6.28702   KNOTS          DBETTA=    51.55513  DEGREES
SUCCESSFUL CONVERGENCE
ITER  =  4
         BOAT SPEED=     4.77718  KNOTS          HEEL ANGLE=     6.77472  DEGREES
         LEEWAY ANGLE=     0.66322  DEGREES          RUDDER ANGLE=     0.66225DEGREES
         ------------------------------------------------
         ------------------------------------------------
         KVT=      5.00000   KNOTS          DGAMMA=    80.00000  DEGREES
         KVA=      7.53630   KNOTS          DBETTA=    40.79662  DEGREES
SUCCESSFUL CONVERGENCE
ITER  =  3
         BOAT SPEED=     4.83670  KNOTS          HEEL ANGLE=    10.99982  DEGREES
         LEEWAY ANGLE=     0.97314  DEGREES          RUDDER ANGLE=     1.19417DEGREES
         ------------------------------------------------
         ------------------------------------------------
         KVT=      5.00000   KNOTS          DGAMMA=    60.00000  DEGREES
         KVA=      8.15836   KNOTS          DBETTA=    32.05679  DEGREES
SUCCESSFUL CONVERGENCE
ITER  =  3
         BOAT SPEED=     4.41289  KNOTS          HEEL ANGLE=    13.76510  DEGREES
         LEEWAY ANGLE=     1.54980  DEGREES          RUDDER ANGLE=     1.49892DEGREES
         ------------------------------------------------
```

```
        ------------------------------------------------
        KVT=      5.00000  KNOTS          DGAMMA=    45.00000  DEGREES
        KVA=      7.97554  KNOTS          DBETTA=    26.31445  DEGREES
SUCCESSFUL CONVERGENCE
ITER    =   4
        BOAT SPEED=     3.61350  KNOTS          HEEL ANGLE=    13.57446  DEGREES
        LEEWAY ANGLE=     2.49007  DEGREES          RUDDER ANGLE=     1.52272DEGREES
        ------------------------------------------------
        ------------------------------------------------
        KVT=      5.00000  KNOTS          DGAMMA=    35.00000  DEGREES
        KVA=      7.44743  KNOTS          DBETTA=    22.64893  DEGREES
SUCCESSFUL CONVERGENCE
ITER    =   4
        BOAT SPEED=     2.77712  KNOTS          HEEL ANGLE=    12.01359  DEGREES
        LEEWAY ANGLE=     3.95441  DEGREES          RUDDER ANGLE=     1.51425DEGREES
        ------------------------------------------------
        ------------------------------------------------
        KVT=      7.50000  KNOTS          DGAMMA=   180.00000  DEGREES
        KVA=      3.76861  KNOTS          DBETTA=   179.99980  DEGREES
SUCCESSFUL CONVERGENCE
ITER    =   4
        BOAT SPEED=     3.73182  KNOTS          HEEL ANGLE=     0.00292  DEGREES
        LEEWAY ANGLE=    -0.00395  DEGREES          RUDDER ANGLE=     0.01209DEGREES
        ------------------------------------------------
        ------------------------------------------------
        KVT=      7.50000  KNOTS          DGAMMA=   160.00000  DEGREES
        KVA=      3.97397  KNOTS          DBETTA=   139.79790  DEGREES
SUCCESSFUL CONVERGENCE
```

```
ITER   =   2
          BOAT SPEED=     4.01511  KNOTS          HEEL ANGLE=     0.05447  DEGREES
          LEEWAY ANGLE=     0.01904  DEGREES          RUDDER ANGLE=     0.01699DEGREES
          ------------------------------------------
          ------------------------------------------
          KVT=     7.50000   KNOTS          DGAMMA=   140.00000  DEGREES
          KVA=     4.91665   KNOTS          DBETTA=   101.32590  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   4
          BOAT SPEED=     4.77972  KNOTS          HEEL ANGLE=     0.86860  DEGREES
          LEEWAY ANGLE=     0.13080  DEGREES          RUDDER ANGLE=     0.05872DEGREES
          ------------------------------------------
          ------------------------------------------
          KVT=     7.50000   KNOTS          DGAMMA=   120.00000  DEGREES
          KVA=     6.75196   KNOTS          DBETTA=    74.14812  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   4
          BOAT SPEED=     5.59371  KNOTS          HEEL ANGLE=     4.78607  DEGREES
          LEEWAY ANGLE=     0.28889  DEGREES          RUDDER ANGLE=     0.56792DEGREES
          ------------------------------------------
          ------------------------------------------
          KVT=     7.50000   KNOTS          DGAMMA=   100.00000  DEGREES
          KVA=     8.77144   KNOTS          DBETTA=    57.35799  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   4
          BOAT SPEED=     6.03347  KNOTS          HEEL ANGLE=    11.95808  DEGREES
          LEEWAY ANGLE=     0.35957  DEGREES          RUDDER ANGLE=     1.76328DEGREES
          ------------------------------------------
```

```
        ------------------------------------------------

        KVT=      7.50000   KNOTS          DGAMMA=    80.00000   DEGREES

        KVA=     10.36289   KNOTS          DBETTA=    45.45833   DEGREES

SUCCESSFUL CONVERGENCE

ITER    =    3

        BOAT SPEED=     5.96515   KNOTS          HEEL ANGLE=     19.44243   DEGREES

        LEEWAY ANGLE=      0.50853   DEGREES          RUDDER ANGLE=     3.10435DEGREES

        ------------------------------------------------

        ------------------------------------------------

        KVT=      7.50000   KNOTS          DGAMMA=    60.00000   DEGREES

        KVA=     11.17890   KNOTS          DBETTA=    35.52245   DEGREES

SUCCESSFUL CONVERGENCE

ITER    =    4

        BOAT SPEED=     5.34839   KNOTS          HEEL ANGLE=     24.68011   DEGREES

        LEEWAY ANGLE=      1.17645   DEGREES          RUDDER ANGLE=     3.94630DEGREES

        ------------------------------------------------

        ------------------------------------------------

        KVT=      7.50000   KNOTS          DGAMMA=    45.00000   DEGREES

        KVA=     11.17072   KNOTS          DBETTA=    28.34309   DEGREES

SUCCESSFUL CONVERGENCE

ITER    =    4

        BOAT SPEED=     4.52826   KNOTS          HEEL ANGLE=     25.90138   DEGREES

        LEEWAY ANGLE=      2.44491   DEGREES          RUDDER ANGLE=     3.91684DEGREES

        ------------------------------------------------

        ------------------------------------------------

        KVT=      7.50000   KNOTS          DGAMMA=    35.00000   DEGREES

        KVA=     10.53564   KNOTS          DBETTA=    24.09880   DEGREES

SUCCESSFUL CONVERGENCE
```

```
        ------------------------------------------------

        KVT=      7.50000  KNOTS          DGAMMA=    80.00000  DEGREES

        KVA=     10.36289  KNOTS          DBETTA=    45.45833  DEGREES

SUCCESSFUL CONVERGENCE

ITER    =    3

        BOAT SPEED=     5.96515  KNOTS          HEEL ANGLE=    19.44243  DEGREES

        LEEWAY ANGLE=     0.50853  DEGREES          RUDDER ANGLE=     3.10435DEGREES

        ------------------------------------------------

        ------------------------------------------------

        KVT=      7.50000  KNOTS          DGAMMA=    60.00000  DEGREES

        KVA=     11.17890  KNOTS          DBETTA=    35.52245  DEGREES

SUCCESSFUL CONVERGENCE

ITER    =    4

        BOAT SPEED=     5.34839  KNOTS          HEEL ANGLE=    24.68011  DEGREES

        LEEWAY ANGLE=     1.17645  DEGREES          RUDDER ANGLE=     3.94630DEGREES

        ------------------------------------------------

        ------------------------------------------------

        KVT=      7.50000  KNOTS          DGAMMA=    45.0000C  DEGREES

        KVA=     11.17072  KNOTS          DBETTA=    28.34309  DEGREES

SUCCESSFUL CONVERGENCE

ITER    =    4

        BOAT SPEED=     4.52826  KNOTS          HEEL ANGLE=    25.90138  DEGREES

        LEEWAY ANGLE=     2.44491  DEGREES          RUDDER ANGLE=     3.91684DEGREES

        ------------------------------------------------

        ------------------------------------------------

        KVT=      7.50000  KNOTS          DGAMMA=    35.00000  DEGREES

        KVA=     10.53564  KNOTS          DBETTA=    24.09880  DEGREES

SUCCESSFUL CONVERGENCE
```

```
ITER    =   4
          BOAT SPEED=     3.47308  KNOTS          HEEL ANGLE=    23.64365  DEGREES
          LEEWAY ANGLE=     4.47674  DEGREES         RUDDER ANGLE=     4.04634DEGREES
          -----------------------------------------------
          -----------------------------------------------
          KVT=     10.00000   KNOTS          DGAMMA=   180.00000  DEGREES
          KVA=      5.16953   KNOTS          DBETTA=   179.99990  DEGREES
SUCCESSFUL CONVERGENCE
ITER    =   4
          BOAT SPEED=     4.83037  KNOTS          HEEL ANGLE=     0.00542  DEGREES
          LEEWAY ANGLE=    -0.00401  DEGREES         RUDDER ANGLE=     0.01243DEGREES
          -----------------------------------------------
          -----------------------------------------------
          KVT=     10.00000   KNOTS          DGAMMA=   160.00000  DEGREES
          KVA=      5.48727   KNOTS          DBETTA=   141.44290  DEGREES
SUCCESSFUL CONVERGENCE
ITER    =   3
          BOAT SPEED=     5.10574  KNOTS          HEEL ANGLE=     0.08175  DEGREES
          LEEWAY ANGLE=     0.01586  DEGREES         RUDDER ANGLE=     0.02775DEGREES
          -----------------------------------------------
          -----------------------------------------------
          KVT=     10.00000   KNOTS          DGAMMA=   140.00000  DEGREES
          KVA=      6.67725   KNOTS          DBETTA=   105.70840  DEGREES
SUCCESSFUL CONVERGENCE
ITER    =   3
          BOAT SPEED=     5.85245  KNOTS          HEEL ANGLE=     1.40412  DEGREES
          LEEWAY ANGLE=     0.10124  DEGREES         RUDDER ANGLE=     0.17259DEGREES
          -----------------------------------------------
```

```
        --------------------------------------------
        KVT=     10.00000  KNOTS          DGAMMA=   120.00000  DEGREES
        KVA=      8.79442  KNOTS          DBETTA=    79.97887  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   8
        BOAT SPEED=     6.52654  KNOTS          HEEL ANGLE=     6.55590  DEGREES
        LEEWAY ANGLE=     0.10446  DEGREES          RUDDER ANGLE=     1.10168DEGREES
        --------------------------------------------
        --------------------------------------------
        KVT=     10.00000  KNOTS          DGAMMA=   100.00000  DEGREES
        KVA=     10.96851  KNOTS          DBETTA=    63.87666  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   4
        BOAT SPEED=     6.56510  KNOTS          HEEL ANGLE=    16.25755  DEGREES
        LEEWAY ANGLE=    -0.16120  DEGREES          RUDDER ANGLE=     3.55508DEGREES
        --------------------------------------------
        --------------------------------------------
        KVT=     10.00000  KNOTS          DGAMMA=    80.00000  DEGREES
        KVA=     12.85525  KNOTS          DBETTA=    50.00259  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   3
        BOAT SPEED=     6.52192  KNOTS          HEEL ANGLE=    27.24298  DEGREES
        LEEWAY ANGLE=    -0.33663  DEGREES          RUDDER ANGLE=     5.98752DEGREES
        --------------------------------------------
        --------------------------------------------
        KVT=     10.00000  KNOTS          DGAMMA=    60.00000  DEGREES
        KVA=     13.80846  KNOTS          DBETTA=    38.84161  DEGREES
SUCCESSFUL CONVERGENCE
```

```
ITER   =   5
          BOAT SPEED=     5.75414  KNOTS          HEEL ANGLE=    35.06798  DEGREES
          LEEWAY ANGLE=     0.37846  DEGREES         RUDDER ANGLE=    7.60845DEGREES
          --------------------------------------------

          --------------------------------------------
          KVT=     10.00000   KNOTS           DGAMMA=    45.00000  DEGREES
          KVA=     13.68682   KNOTS           DBETTA=    31.10666  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   4
          BOAT SPEED=     4.64757  KNOTS          HEEL ANGLE=    37.00015  DEGREES
          LEEWAY ANGLE=     2.45695  DEGREES         RUDDER ANGLE=    8.19907DEGREES
          --------------------------------------------

          --------------------------------------------
          KVT=     10.00000   KNOTS           DGAMMA=    35.00000  DEGREES
          KVA=     12.49910   KNOTS           DBETTA=    27.31577  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   7
          BOAT SPEED=     2.91379  KNOTS          HEEL ANGLE=    32.23439  DEGREES
          LEEWAY ANGLE=     7.24246  DEGREES         RUDDER ANGLE=    13.07657DEGREES
          --------------------------------------------
```

```
ITMAX  =        10
IPRINT =         0
N      =         4
EPS1   =         1.0D-10
EPS2   = ,       1.0E-02
          XOLD(1)...XOLD( 4)

   1.000000E 01    0.000000E-01    0.000000E-01    0.000000E-01
         SOLUTIONS ARE COMPUTED FOR THE FOLLOWING TRUE WIND SPEEDS (KNOTS)

  5.00000   7.50000  10.00000
         SOLUTIONS ARE COMPUTED FOR THE FOLLOWING TRUE WIND ANGLES FOR EACH WIND SPEED

180.00000 160.00000 140.00000 120.00000 100.00000  80.00000  60.00000  45.00000  35.00000
```

```
LWL=    30.00000
DISPLACEMENT TO LENGTH=   225.00000
PRISMATIC=     0.53000
L/B=     14.76796
LCB=     53.00000
KINEMATIC VISCOSITY=   0.1279100E-04
DENSITY OF WATER=      1.99000
MEAN KEEL CHORD=      6.90000
MEAN RUDDER CHORD=      1.87537
ROUGHNESS AND CORRELATION FACTOR=     0.00000
FAIRBODY WETTED SURFACE=   215.25700
KEEL WETTED SURFACE=    63.01086
RUDDER WETTED SURFACE=    19.99725
TOTAL WETTED SURFACE=   298.26480
BWL /LWL=     0.30276
SAIL AREA=   900.00000
HEELING MOMENT ARM=    23.99541
PITCHING MOMENT ARM=    26.35417
DISTANCE FROM FP TO RUDDER STOCK=    28.49998
CLOSE HAULED LIFT COEF=      1.25000
RIG FORM DRAG COEF=     0.40000
CROSS SECTIONAL AREA OF RIG=    26.97595
SAIL FRICTIONAL COEF=      0.04000
INDUCED DRAG COEF / LIFT COEF**2=     0.13030
LATERAL AREA OF HULL=   121.29310
TRANSVERSE AREA OF HULL=    37.06689
DISTANCE BETWEEN CE SAIL AND FP=    13.93835
OFF WIND DRAG COEF=     0.00000
```

```
OFF WIND LIFT COEF=     0.00000
DISTANCE BETWEEN CE HULL AND FP=    13.93835
DENSITY OF AIR=     0.00238
HORIZONTAL DISTANCE FROM MAXIMUM DRAFT OF PROFILE MEASUREMENTS
0.00000   3.00000   6.00000   9.00000  16.50000
PROFILE MEASUREMENTS
6.38000   6.25240   1.92985   1.42200   0.00000
DRAFT OF CANOE BODY / TOTAL DRAFT=     0.31841
HYDRODYNAMIC LIFT COEF=     0.11209
LAMDA X LIFT / DRAG=     2.04978
DISTANCE FROM LWL TO VERTICAL HYDRO CE=     2.12454
*** RUDDER SECTION ***
DISTANCE FROM LWL TO ROOT CHORD=     0.40628
ROOT CHORD=     2.25000
TIP CHORD=     1.50075
RUDDER SPAN=     5.07766
SWEEP ANGLE=     0.00000
ASPECT RATIO=     5.41509
MEAN CHORD=     1.87537
DCL / DALPHA=     0.07057
CROSS FLOW DRAG COEF=     0.56024
DCM / DCL=     0.21654
FORM DRAG COEF=     0.00650
RUDDER AREA=     9.52251
*** RIGHTING MOMENT SECTION ***
METACENTRIC HEIGHT / LWL=     0.11534
FROUDE NUMBER RIGHTING MOMENT COEF=   0.6089515E-04
```

```
NON LINEAR HYDROSTATIC COEF=   0.3767873E-03
DISTANCE BELOW LWL TO CG=     0.75000
*** SAIL SECTION ***
I=    47.17540          J=     15.72513
P=    42.88670          E=     14.29557
AR FOR TRI=     3.00000          AR MAIN=     3.00000
AR SAIL PLAN=     2.47280
CE SAIL PLAN AFT MAST=     0.60041
FORWARD OVERHANG=     2.38720
FREEBOARD=     3.71000
```

```
        ------------------------------------------------

        KVT=     5.00000   KNOTS        DGAMMA=   180.00000  DEGREES

        KVA=     2.36443   KNOTS        DBETTA=   179.99980  DEGREES

SUCCESSFUL CONVERGENCE

ITER   =   5

        BOAT SPEED=     2.63557  KNOTS          HEEL ANGLE=     0.00057  DEGREES

        LEEWAY ANGLE=    -0.00116  DEGREES          RUDDER ANGLE=     0.00433DEGREES
        ------------------------------------------------

        ------------------------------------------------

        KVT=     5.00000   KNOTS        DGAMMA=   160.00000  DEGREES

        KVA=     2.49134   KNOTS        DBETTA=   136.65290  DEGREES

SUCCESSFUL CONVERGENCE

ITER   =   2

        BOAT SPEED=     2.88579  KNOTS          HEEL ANGLE=     0.01554  DEGREES

        LEEWAY ANGLE=     0.02C97  DEGREES          RUDDER ANGLE=     0.00262DEGREES
        ------------------------------------------------
NO CONVERGENCE
        KVT=     5.00000   KNOTS        DGAMMA=   140.00000  DEGREES

        KVA=     3.21880   KNOTS        DBETTA=    93.15175  DEGREES

SUCCESSFUL CONVERGENCE

ITER   =  11

        BOAT SPEED=     3.65326  KNOTS          HEEL ANGLE=     0.24055  DEGREES

        LEEWAY ANGLE=     0.13603  DEGREES          RUDDER ANGLE=     0.02736DEGREES
        ------------------------------------------------

        KVT=     5.00000   KNOTS        DGAMMA=   120.00000  DEGREES

        KVA=     4.83768   KNOTS        DBETTA=    63.51920  DEGREES

SUCCESSFUL CONVERGENCE
```

```
ITER   =   4
          BOAT SPEED=     4.65673  KNOTS            HEEL ANGLE=     1.80063  DEGREES
          LEEWAY ANGLE=     0.54518  DEGREES          RUDDER ANGLE=     0.06240DEGREES
          ------------------------------------------------
          ------------------------------------------------
          KVT=     5.00000  KNOTS          DGAMMA=   100.00000  DEGREES
          KVA=     6.53116  KNOTS          DBETTA=    48.93198  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   4
          BOAT SPEED=     5.15885  KNOTS            HEEL ANGLE=     4.21832  DEGREES
          LEEWAY ANGLE=     0.98065  DEGREES          RUDDER ANGLE=     0.20861DEGREES
          ------------------------------------------------
          ------------------------------------------------
          KVT=     5.00000  KNOTS          DGAMMA=    80.00000  DEGREES
          KVA=     7.84539  KNOTS          DBETTA=    38.87585  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   3
          BOAT SPEED=     5.23940  KNOTS            HEEL ANGLE=     6.78553  DEGREES
          LEEWAY ANGLE=     1.49549  DEGREES          RUDDER ANGLE=     0.37304DEGREES
          ------------------------------------------------
          ------------------------------------------------
          KVT=     5.00000  KNOTS          DGAMMA=    60.00000  DEGREES
          KVA=     8.57841  KNOTS          DBETTA=    30.31612  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   3
          BOAT SPEED=     4.90367  KNOTS            HEEL ANGLE=     8.62737  DEGREES
          LEEWAY ANGLE=     2.18603  DEGREES          RUDDER ANGLE=     0.45298DEGREES
          ------------------------------------------------
```

```
        ----------------------------------------------
        KVT=      5.00000  KNOTS          DGAMMA=    45.00000  DEGREES
        KVA=      8.39471  KNOTS          DBETTA=    24.90797  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   4
        BOAT SPEED=     4.07831  KNOTS          HEEL ANGLE=     8.49245  DEGREES
        LEEWAY ANGLE=     3.16383  DEGREES          RUDDER ANGLE=     0.43054DEGREES
        ----------------------------------------------

        ----------------------------------------------
        KVT=      5.00000  KNOTS          DGAMMA=    35.00000  DEGREES
        KVA=      7.81175  KNOTS          DBETTA=    21.53836  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   4
        BOAT SPEED=     3.17022  KNOTS          HEEL ANGLE=     7.44635  DEGREES
        LEEWAY ANGLE=     4.64773  DEGREES          RUDDER ANGLE=     0.38519DEGREES
        ----------------------------------------------

        ----------------------------------------------
        KVT=      7.50000  KNOTS          DGAMMA=   180.00000  DEGREES
        KVA=      3.59100  KNOTS          DBETTA=   179.99980  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   3
        BOAT SPEED=     3.91276  KNOTS          HEEL ANGLE=     0.00121  DEGREES
        LEEWAY ANGLE=     0.00031  DEGREES          RUDDER ANGLE=     0.00054DEGREES
        ----------------------------------------------

        ----------------------------------------------
        KVT=      7.50000  KNOTS          DGAMMA=   160.00000  DEGREES
        KVA=      3.81778  KNOTS          DBETTA=   137.78660  DEGREES
SUCCESSFUL CONVERGENCE
```

```
ITER   =   2
          BOAT SPEED=      4.21828  KNOTS             HEEL ANGLE=      0.03173  DEGREES
          LEEWAY ANGLE=      0.02009  DEGREES            RUDDER ANGLE=      0.00429DEGREES
          ------------------------------------------------

          ------------------------------------------------
          KVT=       7.50000  KNOTS          DGAMMA=   140.00000  DEGREES
          KVA=       4.86734  KNOTS          DBETTA=    97.92107  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   3
          BOAT SPEED=      5.07021  KNOTS             HEEL ANGLE=      0.49961  DEGREES
          LEEWAY ANGLE=      0.14829  DEGREES            RUDDER ANGLE=      0.01619DEGREES
          ------------------------------------------------

          ------------------------------------------------
          KVT=       7.50000  KNOTS          DGAMMA=   120.00000  DEGREES
          KVA=       6.84121  KNOTS          DBETTA=    71.69914  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   3
          BOAT SPEED=      5.90355  KNOTS             HEEL ANGLE=      2.91928  DEGREES
          LEEWAY ANGLE=      0.51083  DEGREES            RUDDER ANGLE=      0.18288DEGREES
          ------------------------------------------------

          ------------------------------------------------
          KVT=       7.50000  KNOTS          DGAMMA=   100.00000  DEGREES
          KVA=       9.05576  KNOTS          DBETTA=    54.64867  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   3
          BOAT SPEED=      6.53885  KNOTS             HEEL ANGLE=      7.42339  DEGREES
          LEEWAY ANGLE=      0.95508  DEGREES            RUDDER ANGLE=      0.56130DEGREES
          ------------------------------------------------
```

```
        ------------------------------------------
        KVT=     7.50000  KNOTS          DGAMMA=    80.00000  DEGREES
        KVA=    10.80911  KNOTS          DBETTA=    43.10342  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   3
        BOAT SPEED=     6.58876  KNOTS          HEEL ANGLE=    12.24898  DEGREES
        LEEWAY ANGLE=     1.51378  DEGREES          RUDDER ANGLE=     0.99877DEGREES
        ------------------------------------------
        ------------------------------------------
        KVT=     7.50000  KNOTS          DGAMMA=    60.00000  DEGREES
        KVA=    11.74616  KNOTS          DBETTA=    33.57051  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   4
        BOAT SPEED=     6.03699  KNOTS          HEEL ANGLE=    15.68644  DEGREES
        LEEWAY ANGLE=     2.41843  DEGREES          RUDDER ANGLE=     1.26121DEGREES
        ------------------------------------------
        ------------------------------------------
        KVT=     7.50000  KNOTS          DGAMMA=    45.00000  DEGREES
        KVA=    11.81018  KNOTS          DBETTA=    26.68242  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   3
        BOAT SPEED=     5.24840  KNOTS          HEEL ANGLE=    16.53915  DEGREES
        LEEWAY ANGLE=     3.57327  DEGREES          RUDDER ANGLE=     1.20193DEGREES
        ------------------------------------------
        ------------------------------------------
        KVT=     7.50000  KNOTS          DGAMMA=    35.00000  DEGREES
        KVA=    11.24568  KNOTS          DBETTA=    22.49055  DEGREES
SUCCESSFUL CONVERGENCE
```

```
ITER  =  4
        BOAT SPEED=    4.24629  KNOTS          HEEL ANGLE=    15.29806  DEGREES
        LEEWAY ANGLE=     5.22368  DEGREES          RUDDER ANGLE=     1.09039DEGREES
        ------------------------------------------------
        ------------------------------------------------
        KVT=    10.00000  KNOTS          DGAMMA=   180.00000  DEGREES
        KVA=     4.96443  KNOTS          DBETTA=   179.99980  DEGREES
SUCCESSFUL CONVERGENCE
ITER  =  3
        BOAT SPEED=    5.03554  KNOTS          HEEL ANGLE=     0.00228  DEGREES
        LEEWAY ANGLE=    -0.00046  DEGREES          RUDDER ANGLE=     0.00276DEGREES
        ------------------------------------------------
        ------------------------------------------------
        KVT=    10.00000  KNOTS          DGAMMA=   160.00000  DEGREES
        KVA=     5.32662  KNOTS          DBETTA=   140.05190  DEGREES
SUCCESSFUL CONVERGENCE
ITER  =  3
        BOAT SPEED=    5.31337  KNOTS          HEEL ANGLE=     0.04492  DEGREES
        LEEWAY ANGLE=     0.01862  DEGREES          RUDDER ANGLE=     0.00731DEGREES
        ------------------------------------------------
        ------------------------------------------------
        KVT=    10.00000  KNOTS          DGAMMA=   140.00000  DEGREES
        KVA=     6.61337  KNOTS          DBETTA=   103.60230  DEGREES
SUCCESSFUL CONVERGENCE
ITER  =  3
        BOAT SPEED=    6.10511  KNOTS          HEEL ANGLE=     0.78191  DEGREES
        LEEWAY ANGLE=     0.14843  DEGREES          RUDDER ANGLE=     0.04948DEGREES
        ------------------------------------------------
```

KVT= 10.00000 KNOTS DGAMMA= 120.00000 DEGREES

KVA= 8.88492 KNOTS DBETTA= 77.08757 DEGREES

SUCCESSFUL CONVERGENCE

ITER = 9

BOAT SPEED= 6.98967 KNOTS HEEL ANGLE= 4.09941 DEGREES

LEEWAY ANGLE= 0.45382 DEGREES RUDDER ANGLE= 0.35205DEGREES

KVT= 10.00000 KNOTS DGAMMA= 100.00000 DEGREES

KVA= 11.29239 KNOTS DBETTA= 60.70335 DEGREES

SUCCESSFUL CONVERGENCE

ITER = 8

BOAT SPEED= 7.26006 KNOTS HEEL ANGLE= 10.27948 DEGREES

LEEWAY ANGLE= 0.89417 DEGREES RUDDER ANGLE= 1.12386DEGREES

KVT= 10.00000 KNOTS DGAMMA= 80.00000 DEGREES

KVA= 13.33439 KNOTS DBETTA= 47.60773 DEGREES

SUCCESSFUL CONVERGENCE

ITER = 3

BOAT SPEED= 7.25288 KNOTS HEEL ANGLE= 17.39751 DEGREES

LEEWAY ANGLE= 1.48165 DEGREES RUDDER ANGLE= 2.01724DEGREES

KVT= 10.00000 KNOTS DGAMMA= 60.00000 DEGREES

KVA= 14.65839 KNOTS DBETTA= 36.21414 DEGREES

SUCCESSFUL CONVERGENCE

```
ITER   =   5
          BOAT SPEED=      6.82657  KNOTS          HEEL ANGLE=    23.44096  DEGREES
          LEEWAY ANGLE=     2.54576  DEGREES          RUDDER ANGLE=     2.50608DEGREES
          ----------------------------------------------------
          ----------------------------------------------------
          KVT=    10.00000  KNOTS          DGAMMA=    45.00000  DEGREES
          KVA=    14.70452  KNOTS          DBETTA=    28.74271  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   4
          BOAT SPEED=      5.82167  KNOTS          HEEL ANGLE=    24.95070  DEGREES
          LEEWAY ANGLE=     4.21530  DEGREES          RUDDER ANGLE=     2.51346DEGREES
          ----------------------------------------------------
          ----------------------------------------------------
          KVT=    10.00000  KNOTS          DGAMMA=    35.00000  DEGREES
          KVA=    14.17157  KNOTS          DBETTA=    23.87466  DEGREES
SUCCESSFUL CONVERGENCE
ITER   =   4
          BOAT SPEED=      4.76666  KNOTS          HEEL ANGLE=    23.88383  DEGREES
          LEEWAY ANGLE=     6.43894  DEGREES          RUDDER ANGLE=     2.34001DEGREES
          ----------------------------------------------------
```